行政院國家科學委員會專題研究計畫成果報告
強健嚴格正實系統之分析與設計
Analysis and Design of Robust Strict Positive Real Systems

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主持人:鄭智湧 海洋大學電機系

Abstract

本計畫的主要目的，在於利用值集的幾何概念，提供一種統一的方法來研究參數不確定系統的嚴格正實性質。其中包括系統嚴格正實的分析與設計。分析的對象分別爲區間、擬似線性及多重擬似線性不確定系統的強健嚴格正實性質。而設計則利用值集不同的特性，採用簡單的補償方法設計一控制器，使得參數不確定系統在補償後成爲一強健嚴格正實之系統。

The main theme of this proposal is to provide a unified approach, based on the geometric concept of value set, for studying strict positive realness of parametric uncertainty systems. The discussion is divided into analysis and design parts. In the first part, interval, affine linear and multiaffine uncertainty systems are analyzed respectively. In the second part, controllers are then designed repectively in order to render the compensated systems strict positive real.

1 Introduction

The issue of robustness in control system designs has been one of the main research interests over the past fifteen years. In the early stage, the small gain theorem is used as the principal tools for modelling plant uncertainty. H_∞ control timely supplied a powerful tool for robust control system designs. This approach is based on shaping of singular values, which is essentially a gain concept.

Gradually, the attention extends from the concept of "gain" to the concept of "phase", which is closely related to the concept of positive realness. A real rational function $G(s) = \frac{N(s)}{D(s)}$ is said to be *strict positive real* (SPR) if:

- $G(s)$ is analytic in the closed right half complex plane;
- $Re[G(s)] > 0$, for $Re[s] > 0$.

or

- $D(s)$ is Hurwitz;
- $Re[G(j\omega)] > 0, \forall\omega \in I\!R$.

Therefore, for an SISO system, we have the very important *phase condition* that the polar plot of a SPR transfer function must lie in the right half of the complex plane. That is,

$$-\frac{\pi}{2} < |arg[N(j\omega)] - arg[D(j\omega)]| < \frac{\pi}{2}, \ \forall\omega \in I\!R.$$

The strict positive realness can be thought of as a counterpart of small gain concept. Small gain theorem deals with the robust stability of a feedback loop consisting of a nominal system and a norm-bounded uncertainty block and it implies that the tolerance of uncertainty will increase if we can reduce the norm of the nominal system. Similar to the small gain theorem, the robust stability of a feedback loop consisting of a nominal system and a positive real uncertainty block will be retained if the nominal system is strictly positive real. The dual concept of small gain and positive real is shown in Figure 1. In the small gain case, system gain is restricted inside a finite number, say 1, while in the positive real case, system phase is restricted within $\pm 90^o$.

In this report, we model the uncertain system as a real rational function family

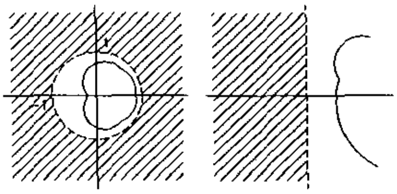

Figure 1: small gain versus positive real

$$G(s) = \frac{N(s,a)}{D(s,b)}$$

where

$$N(s,a) = \sum_{i=0}^{m} a_i s^i, \quad D(s,b) = \sum_{i=0}^{n} b_i s^i$$

a_i, b_i are real uncertain coefficients.

The uncertain parameters could be of independent, affine linear or multilinear structures, which will be defined in later sections. And a given family of transfer functions is said to be robust SPR if every member of the family is SPR.

2 Robust SPR Analysis

We begin with the case of systems with multiaffine uncertainty. Indeed, the affine linear and interval cases will be one of its special cases.

2.1 Multiaffine uncercertainty

Multiaffine cases can arise frequently in uncertain system descriptions.

1. In frequency domain description, for example, we only roughly know the two dominant poles location of a transfer function

$$G(s) = \frac{1}{(s+q_1)(s+q_2)}$$

   and we can describe q_1 and q_2 to be in an interval to account for the uncertainty. Therefore we can rewrite it as

$$G(s) = \frac{1}{s^2 + (q_1+q_2)s + q_1 q_2}$$

   where it becomes a multilinear case.

2. In time domain description, for example, we represent a two state system as

$$\begin{aligned} \dot{x} &= Ax + bu \\ &= \begin{bmatrix} q_1 & q_2 \\ q_3 & q_4 \end{bmatrix} x + \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} u. \end{aligned}$$

   To accout for the uncertainty, which could be resulted from modelling inaccuracies, we allow q_1, q_2, q_3 and q_4 to lie in an interval. When computing its characteristic polynomial

$$\det(sI - A) = s^2 - (q_1+q_4)s + (q_1 q_4 - q_2 q_3),$$

   the multilinear structure came out naturally due to the operation of determinant.

Theorem 1: A proper transfer function family $\mathcal{G}_{multi}$ is robust SPR if and only if the following conditions are satisfied:

- $Re[G(0)] > 0, \forall G(s) \in \mathcal{G}_{multi}$
- $\mathcal{N}_{multi}$ and $\mathcal{D}_{multi}$ contain at least one Hurwitz stable polynomial, respectively
- the worst case phase should lie within $\pm\frac{\pi}{2}$.

The third condition in the multilinear case can easily be checked by only vertices information.

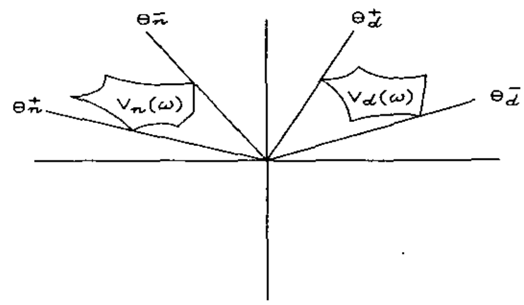

Figure 2: Value sets for multilinear system

Example: Consider the following family $G(s)$ of multilinear system whose generic element is given by

$$G(s) = \frac{s^2 + (q_1+q_2)s + q_1 q_2}{q_4 s^3 + (q_3 + q_4 + q_4 q_5)s^2 + (q_3 q_5 + q_4 q_5 + q_3)s + q_3 q_5}$$

and $q_1 \in [0.5, 1]$, $q_2 \in [0.5, 1]$, $q_3 \in [1, 2]$, $q_4 \in [1, 2]$, $q_5 \in [1, 2]$. Check whether this system is robust SPR or not?

Solution:

1. $Re[G(0)] = \frac{q_1 q_2}{q_3 q_5}$. Due to q_1, q_2, q_3, q_4 are all positive numbers, $Re[G(0)] > 0$.

2. Let $q_1 = 0.4$, $q_2 = 0.6$, $q_3 = 1.5$, $q_4 = 1.5$, $q_5 = 1.5$, Then the roots of numerator and denominator polynomials are $\{-0.4, -0.6\}$ and $\{-1, -1, -1.5\}$, respectively. So, the numerator and denominator polynomials are both Hurwitz stable.

3. From Figure 3, the third point of Theorem 3 is satisfied.

Therefore, the system is robust SPR.

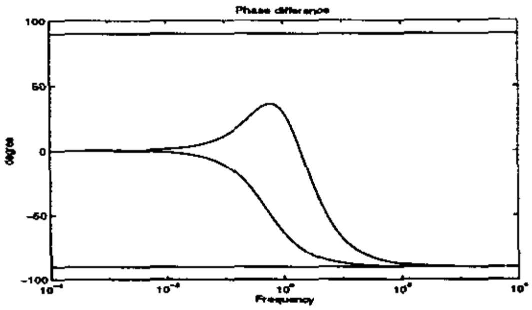

Figure 3: Phase difference

2.2 Affine linear uncercertainty

For systems with affine linear uncertainties, the previous vertex results can still be applied. But with this simplied uncertainty structure, we can easily distinguish the exposed vertices by procedure in [9]. Therefore, the testing set can further be reduced to exposed vertices.

2.3 Interval uncercertainty

In case we are dealing with interval systems, the testing set can be explicitly written as follows.

Theorem 2: An interval family $\mathcal{G}_I = \frac{N(s,q)}{D(s,r)}$ of proper transfer functions is SPR if and only if the following eight transfer functions are SPR.

$$G_1 := \frac{N_4}{D_1},\quad G_2 := \frac{N_3}{D_1},\quad G_3 := \frac{N_1}{D_4},\quad G_4 := \frac{N_2}{D_4}$$

$$G_5 := \frac{N_1}{D_3},\quad G_6 := \frac{N_2}{D_3},\quad G_7 := \frac{N_4}{D_2},\quad G_8 := \frac{N_3}{D_2}$$

where

$$\begin{aligned} N_1(s) &= q_0^- + q_1^- s + q_2^+ s^2 + q_3^+ s^3 + \cdots \\ N_2(s) &= q_0^+ + q_1^+ s + q_2^- s^2 + q_3^- s^3 + \cdots \\ N_3(s) &= q_0^+ + q_1^- s + q_2^- s^2 + q_3^+ s^3 + \cdots \\ N_4(s) &= q_0^- + q_1^+ s + q_2^+ s^2 + q_3^- s^3 + \cdots \end{aligned}$$

and $D_i(s)$ are similarly defined.

3 Robust SPR Synthesis

Recall that for an SISO system to be strictly positive real, the phase of the transfer function should remain within the range of $\pm 90^o$ for all freqnencies.

3.1 Lead compensation

Recall a lead compensator

$$C(s) = \frac{s+a}{s+b} \quad \text{where} \quad a < b$$

and it was well known that

$$\begin{cases} \omega_{max} &= \sqrt{ab} \\ \theta_{max} &= \sin^{-1} \frac{b-a}{b+a}. \end{cases}$$

Example: lead compensation Now if we are given a system $G(s) = \frac{s+10}{s^2+2s+1}$, we know $G(s)$ is not positive real. Applying the concept of lead compensation, we provide a positive (lead) angle of 50^o at the frequency $\omega = 4$. Solving the equations, we have

$$\begin{cases} a &= 1.456 \\ b &= 10.9887. \end{cases}$$

Therefore, we can check that after this lead compensation, the closed-loop transfer function $\frac{C(s)G(s)}{1+C(s)G(s)}$ is now positive real.

3.2 Interlacing patterns

With the restriction of $\pm 90^o$ on phase angle, we know that a positive real transfer function cannot have two consecutive occurences of poles and zeros. Hence, one way to make a system positive real is to insert some poles and zeros and make the pole/zero become an interlacing pattern.

Note that the interlacing is only a sufficient condition here. Observe that in the previous example, we do not have the pole/zero interlacing pattern, but the closed-loop system is indeed positive real.

Example: real pole/zero Given a nonpassive system $G(s) = \frac{s+3}{s(s+2)}$. If we can insert certain poles and zeros into the original system using a *proper* cascade controller, the resulting system will become a passivie (positive real) one. For example, choose $C(s) = \frac{s+1}{s+4}$ and the resulting system

$$C(s)G(s) = \frac{(s+1)(s+3)}{s(s+2)(s+4)}$$

is positive real.

Example: complex pole/zero This idea of interlacing can also be applied to systems with complex pole/zero. For example, let us first consider the case $G(s) = \frac{s^2+2}{s^2+1}$ whose poles and zeros are on the $j\omega$-axis. Judging from its bode or nyquist plot, $G(s)$ is not positive real. After the cascade compensation with $C(s) = \frac{s}{s^2+3}$, the system

$$C(s)G(s) = \frac{s(s^2+2)}{(s^2+1)(s^2+3)}$$

is positive real.

Sometimes, the complex (not necessarily imaginary) pole/zero interlacing is not obvious. We can project the complex pole/zeros onto the $j\omega$-axis to investigate this property. For example,

$$G(s) = \frac{s(s^2+2)}{(s^2+s+1)(s^2+s+3)}$$

is indeed positive real.

Example: unstable or nonminimum phase But the above-mentioned trick will face the problems of internal stability in case of unstable or nonminimum phase systems. To remedy such situations, we should introduce feedback structure. A very simple example is when $G(s) = \frac{2}{s-1}$. Obviously, it is not positive real. And it is not possible to compensate using a cascade controller due to right half plane pole-zero cancellation. But just with a unity-feedback wrapped around, the closed-loop system becomes $\frac{G(s)}{1+G(s)} = \frac{\frac{2}{s-1}}{1+\frac{2}{s-1}} = \frac{2}{s+1}$, which is positive real.

Indeed, such interlacing compensation techniques are closely related to the interlacing property of the seminal Kharitonov theorem.

4 Conclusion

Positive realness is an important and fundamental notion in network and system theory. In this report, the problem of robust strict positive realness for uncertain systems defined by interval, affine linear and multilinear were discussed. Under a unified geometric framework, we can conclude that to guarantee the robust SPR property, we need only check some extreme points of the whole family of systems. Furthermore, in certain cases, simple controllers can be designed to render the system robust SPR.

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