行政院國家科學委員會專題研究計畫成果報告
線性時變系統回授穩定及 H^∞ 控制及混合 H^2/H^∞ 控制問題之研究
Feedback Stabilization, H^∞ Control and Mixed H^2/H^∞ Control for Linear Time-Varying Systems

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主持人:容志輝 國立臺灣海洋大學電機系

摘要

本計畫目的在研究有限維度線性時變系統之回授穩定，H^∞ 控制及混合 H^2/H^∞ 控制問題。對一線性時變受控體，吾人將推導其所有內部穩定控制器及所有 H^∞ 控制器之狀態空間表示式。本計畫同時採用Nash局論觀點求得無限時區混合 H^2/H^∞ 控制問題之狀態回授及靜態輸出回授解。

關鍵詞:Youla參數化，H^∞ 控制，混合 H^2/H^∞ 控制，Nash局論，線性時變系統。

Abstract

The project is concerned with feedback stabilization, H^∞ control and mixed H^2/H^∞ control for finite-dimensional linear time-varying systems. In particular, we derive state-space formulas for all exponentially (internally) stabilizing controllers for a given linear time-varying plant. In addition, we also give a state-space characterization of all H^∞ controllers for a given linear time-varying plant. Finally, based on a Nash game approach, we give solutions to the infinite horizon mixed H^2/H^∞ control for linear time-varying systems via both state feedback and static output feedback.

Keywords: Youla parameterization, H^∞ control, mixed H^2/H^∞ control, Nash game, linear time-varying systems.

1 Introduction

This project investigates feedback stabilization, H^∞ control and mixed H^2/H^∞ control for linear time-varying systems. Consider the standard system connection shown in Figure 1, where the plant G is a finite-dimensional linear time-varying system. Here $u \in \mathbb{R}^{m_2}$

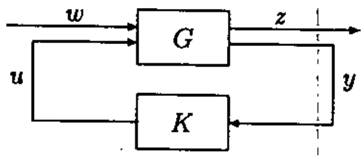

Fig. 1 Standard System Connection

is the control input, $w \in \mathbb{R}^{m_1}$ represents a set of exogenous inputs, $z \in \mathbb{R}^{p_1}$ is the controlled output, $y \in \mathbb{R}^{p_2}$ is the measured variable, and the matrices $A(\cdot)$, $B_i(\cdot)$, $C_j(\cdot)$, $D_{ij}(\cdot)$ are assumed to be all bounded functions of t of appropriate dimensions.

The control action is to be provided by a controller K. Only finite-dimensional linear time-varying controllers are considered in this project.

1.1 Feedback Stabilization

In the first part (Section 2) of this report, we will study the feedback stabilization problem for linear time-varying systems. The feedback stabilization problem for the plant G shown in Figure 1 is the problem of parameterizing *all* controllers K that exponentially (internally) stabilize the resulting closed-loop system.

For the linear time-invariant case, the parameterization for all internally stabilizing controllers was first introduced by Youla et al. [20] [21] using the coprime factorization technique. Lu [12] was the first to apply a state-space approach to the Youla parameterization of stabilizing controllers for linear and nonlinear time-invariant systems using the technique developed in [7] and [13].

Motivation for studying the Youla parameterization for time-varying system comes from the facts that time-varying system stability is an important area, and that many results that are relatively easy to establish in the time-invariance case become much more difficult and interesting in the general time-varying case. Furthermore, the results of this part will be used in the sequel.

1.2 H^∞ Control

Robustness of control systems to disturbances and uncertainties has always been the central issue in feedback control. One of the major new developments in robust control theory during the last two decades in the control community has been the introduction of state-space solutions to the standard H^∞ control problem for linear time-invariant case in terms of the solutions to two Riccati equations [7]. A parameterization of all H^∞-(sub)optimal controllers was also given in [7].

The state-space approach is particularly natural and appealing for the H^∞ control problem of linear time-varying systems; see e.g. [17], [18], [11] and [15]. In particular, under the standard assumptions as in [7], Ravi *et al.* [15] obtained necessary and sufficient conditions for the existence of solutions and constructed one controller (namely the central controller) to the H^∞ control problem of linear time-varying systems for the infinite horizon case.

Regarding design, obtaining an H^∞-(sub)optimal controller is not always a sole control system design goal in practice. Besides the closed-loop internal stability and closed-loop H^∞-norm requirement, several other secondary considerations also come into play in the final stages of controller design. These considerations include flexibility of assigning closed-loop eigenvalues at the desired locations in the negative half complex plane, linear-quadratic(LQ) performance measure (namely, H^2 norm of a subsystem transfer function of the given plant), and bandwidth requirements on a controller, etc. In order to make it viable to select an H^∞-(sub)optimal controller which also meets some such secondary considerations, it is important to parameterize and characterize the set of all H^∞ controllers. This is important both from an engineering as well as from a theoretical viewpoint. After obtaining the parameterization and characterization of the set of all H^∞ controllers, one can select a particular controller from this set which also satisfies some given secondary considerations.

As motivated by the above discussion, the purpose of the second part (Section 3) of this report is to give a parameterization of all H^∞ controllers for a linear time-varying system. While the underlying idea in this part is essentially the same idea used in [7] to parameterize the set of all H^∞-(sub)optimal controllers in the time-invariant case, the proofs of these results are not so easy to generalize and actually involve significant technical difficulties in the present time-varying context.

1.3 Mixed H^2/H^∞ Control

As mentioned before, designing an H^∞ controller is not always a sole control system design goal. In fact, multi-objective controller synthesis has been an active research area in the control community during the last decade; see, e.g., [2], [4], [5], [8], [9], [14] and [16]. In particular, mixed H^2/H^∞ control problem has received a great deal of attention and several authors have investigated the mixed H^2/H^∞ control problem by means of different approaches. In the work of [2], the authors provide a solution to mixed H^2/H^∞ control problem by designing the LQG control subject to a constraint on H^∞ disturbance attenuation. The H^∞-constrained gains are characterized by a coupled system of three modified Riccati equations.

The work of [2] is extended in [8], where another mixed H^2/H^∞ problem is addressed. The system considered in [8] is dual to the Bernstein-Haddad setup [2], and the outcome of [8] is a formula for a mixed H^2/H^∞ controller characterized in terms of a pair of cross-coupled Riccati equations and a standard H^∞ Riccati equation. It was shown later by Yeh *et al.* [19] that the solution of [2] and the solution presented by Doyle *et al.* [8] are actually dual to each other.

In contrast to the other work in this area, Khargonekar and Rotea [9] examined multi-objective control problems, and developed an algorithmic solution to a mixed H^2/H^∞ control problem based on convex optimization.

Recently Limebeer *et al.* [10] have developed a design methodology for mixed H^2/H^∞ control problem via the solution of a two-player nonzero-sum Nash Game [3]. It is well known that two-player nonzero-sum differential games have two performance criteria [3], and the central idea behind the work of [10] is to use the two performance indices to reflect the H^2 optimization and H^∞ constraint separately. For the finite horizon linear time-varying systems case, it turns out that, under some appropriate assumptions, the solution to the mixed H^2/H^∞ control problem via state feedback is determined by the solution of a pair of cross-coupled Riccati equations [10].

In the last part (Section 4) of the report, we first give a solution to the *infinite horizon* linear time-varying systems case via state feedback building on the work of [10]. It should be noted that the infinite horizon case is more difficult since stability becomes an important issue, whereas in the finite horizon case this is completely absent. In addition, the difficulty is more significant due to time-varying characteristics.

If the state of the plant is not available for measurement, then the feedback law via state feedback cannot be directly used. For this reason, we next extend the state feedback results of [10] to output feedback case for infinite horizon linear time-varying systems. Only *static* output feedback case will be considered in this report. This also extends some results in [6], where the static output feedback case for infinite horizon linear time-invariant systems was extensively studied. The difficulties associated with dynamic feedback strategies are addressed by example in [1]. It should be pointed out that the output feedback case is more involved. This is because in the output feedback case the solution may not be unique even we restrict the admissible strategy set to that of linear memoryless ones, whereas in the state feedback case there is a unique global Nash solution if we force both players to use linear, memoryless feedback controls.

2 Feedback Stabilization

We are now in the position to state a version of Youla parameterization for linear time-varying case. In what follows, we denote the feedback connection of Fig. 1 by $LFT(G, K)$.

Theorem 1: Consider the linear time-varying system

$$\sum := \begin{cases} \dot{x}(t) &= A(t)x(t) + B(t)u(t) \\ y(t) &= C(t)x(t) + D(t)u(t) \end{cases} \quad t \in \mathbb{R}^+ := [0, \infty) \tag{1}$$

Suppose that $\sum$ is stabilizable and detectable. Let $F(\cdot)$ and $L(\cdot)$ be bounded functions such that $A + BF$ and $A + LC$ are exponentially stable. Then all finite-dimensional linear time-varying (FDLTV) controllers that exponentially stabilize $\sum$ can be parameterized as $K = LFT(J, Q)$, where

$$J := \begin{cases} \dot{\xi}(t) &= (A + BF + LC + LDF)(t)\xi(t) \\ &\quad -L(t)y(t) + (B + LD)(t)p(t) \\ u(t) &= F(t)\xi(t) + p(t) \\ d(t) &= -(C + DF)(t)\xi(t) + y(t) - D(t)p(t) \end{cases}$$

with the free system

$$Q := \begin{cases} \dot{q}(t) &= A_q(t)q(t) + B_q(t)d(t) \\ p(t) &= C_q(t)q(t) + D_q(t)d(t) \end{cases}$$

exponentially stable, and $I + D(t)D_q(t)$ invertible for all $t \in \mathbb{R}^+$.

Proof: See [23].

3 H^∞ Controller Parameterization

3.1 Problem Formulation and Preliminaries

Consider the standard system connection shown in Fig. 1. Suppose that the plant G is an FDLTV system described by equations of the form

$$G := \begin{cases} \dot{x}(t) &= A(t)x(t) + B_1(t)w(t) + B_2(t)u(t) \\ z(t) &= C_1(t)x(t) + D_{12}(t)u(t) \\ y(t) &= C_2(t)x(t) + D_{21}(t)w(t) \\ x(0) &= 0, \quad t \in \mathbb{R}^+ \end{cases} \tag{2}$$

The standard H^∞ control problem consists of finding (when one exists) an FDLTV controller K that exponentially (internally) stabilizes G and makes $\|T_{zw}\|_\infty < \gamma$. Here T_{zw} represents the closed-loop operator mapping w to z, and the infinity norm $\|\cdot\|_\infty$ is induced by $\mathcal{L}^2$-norm.

We will make the following standard assumptions considered in, e.g., [15], [7].

A1) $D_{12}^T(t)C_1(t) = 0$, $D_{12}^T(t)D_{12}(t) = I$

A2) $B_1(t)D_{21}^T(t) = 0$, $D_{21}(t)D_{21}^T(t) = I$

A3) (A, B_1) is stabilizable and (C_1, A) is detectable.

A4) (A, B_2) is stabilizable and (C_2, A) is detectable.

The following theorem, taken from [15], shows that a solution to the H^∞ control problem exists if and only if a pair of Riccati differential equations admits nonnegative definite stabilizing solutions. The theorem also provides state-space formulae for one solution (i.e., the central controller) to the problem.

Theorem 2: Consider (2) and suppose Assumptions A1)-A4) are satisfied. Then there exists a controller K that exponentially stabilizes G and makes $\|T_{zw}\|_\infty < \gamma$ if and only if

i) there exists a bounded nonnegative definite solution X_∞ to the Riccati equation

$$\begin{aligned} -\dot{X}_\infty(t) &= A^T(t)X_\infty(t) + X_\infty(t)A(t) \\ &\quad + X_\infty(t)(\tfrac{1}{\gamma^2}B_1(t)B_1^T(t) - B_2(t)B_2^T(t))X_\infty(t) \\ &\quad + C_1^T(t)C_1(t) \end{aligned} \tag{3}$$

such that the system $\dot{x}(t) = (A + (\frac{1}{\gamma^2}B_1B_1^T - B_2B_2^T)X_\infty)(t)x(t)$ is exponentially stable, and

ii) there exists a bounded nonnegative definite solution Z_∞ to the Riccati equation

$$\begin{aligned} \dot{Z}_\infty(t) &= A_Z(t)Z_\infty(t) + Z_\infty(t)A_Z^T(t) \\ &\quad + Z_\infty(t)(\tfrac{1}{\gamma^2}X_\infty(t)B_2(t)B_2^T(t)X_\infty(t) - C_2^T(t)C_2(t))Z_\infty(t) \\ &\quad + B_1(t)B_1^T(t), \qquad Z_\infty(0) = 0 \end{aligned} \tag{4}$$

such that the system

$$\dot{x}(t) = (A_Z + Z_\infty(\frac{1}{\gamma^2}X_\infty B_2B_2^TX_\infty - C_2^TC_2))(t)x(t)$$

is exponentially stable, where

$$A_Z(t) := (A + \frac{1}{\gamma^2} B_1 B_1^T X_\infty)(t)$$

If these conditions are met, then one controller solving the problem is given by

$$K := \begin{cases} \dot{\xi}(t) &= (A_Z - B_2 B_2^T X_\infty - Z_\infty C_2^T C_2)(t)\xi(t) \\ & \quad + Z_\infty(t) C_2^T(t) y(t) \\ u(t) &= -B_2^T(t) X_\infty(t) \xi(t) \end{cases}$$

3.2 Main Results

The following theorem, which is the main result of this section, characterizes all FDLTV controllers that exponentially stabilize G and make $\|T_{zw}\|_\infty < \gamma$

Theorem 3: Consider (2) and suppose Assumptions A1)-A4) are satisfied. Suppose Conditions i) and ii) of Theorem 2 hold. Then the set of all FDLTV internally stabilizing controllers such that $\|T_{zw}\|_\infty < \gamma$ can be parameterized as $K = LFT(M, Q)$, where

$$M := \begin{cases} \dot{\xi}(t) &= (A_Z - B_2 B_2^T X_\infty - Z_\infty C_2^T C_2)(t)\xi(t) \\ & \quad + Z_\infty(t) C_2^T(t) y(t) \\ & \quad + (I + \frac{1}{\gamma^2} Z_\infty X_\infty)(t) B_2(t) p(t) \\ u(t) &= -B_2^T(t) X_\infty(t) \xi(t) + p(t) \\ d(t) &= -C_2(t)\xi(t) + y(t) \end{cases}$$

and the free system

$$Q := \begin{cases} \dot{q}(t) &= A_q(t) q(t) + B_q(t) d(t) \\ p(t) &= C_q(t) q(t) + D_q(t) d(t) \end{cases}$$

is exponentially stable with $\|T_{pd}\|_\infty < \gamma$.

Proof: See [23].

4 Mixed H^2/H^∞ Control

4.1 Problem Formulation and Preliminaries

Suppose we are given a finite-dimensional linear time-varying plant G described by (2). The infinite horizon mixed H^2/H^∞ control problem under consideration is concerned with constructing a feedback control law $u^*(x,t)$ such that the resulting closed-loop system is exponentially (internally) stable and satisfies an H^∞ norm constraint, namely $\|T_{zw}\|_\infty < \gamma$. Here T_{zw} is the input-output operator mapping w to z when the control law $u^*(x,t)$ is applied. In addition, we require the control $u^*(x,t)$ to regulate the state $x(t)$ in such a way as to minimize the output energy when the worst-case disturbance $w^*(x,t)$ is applied to the system. The signal $w^*(x,t)$ is worst case in the sense that it achieves the maximum possible energy gain from the disturbance input $w(t)$ to the controlled output $z(t)$.

As in [10], the problem in question can be cast as a two players, nonzero sum, Nash differential game of infinite duration $[0,\infty)$, and two cost functions

$$J_1(w,u) = \int_0^\infty (\gamma^2 w^T(t) w(t) - z^T(t) z(t)) dt \quad (5)$$

$$J_2(w,u) = \int_0^\infty z^T(t) z(t) dt \quad (6)$$

The first can be viewed as an H^∞ criterion, while the second is associated with the H^2 optimization. Thus we wish to find equilibrium strategies u^* and w^* such that

$$J_1(w^*,u^*) \le J_1(w,u^*) \quad (7)$$

$$J_2(w^*,u^*) \le J_2(w^*,u) \quad (8)$$

4.2 State Feedback Case

Theorem 4: Consider (2) with (C_1, A) detectable. Assume Assumption A1) is satisfied. Suppose there exist solutions $P_1(t) \le 0$ and $P_2(t) \ge 0$ satisfying:

$$-\frac{dP_1(t)}{dt} = A^T(t) P_1(t) + P_1(t) A(t) - C_1^T(t) C_1(t) \quad (9)$$

$$-\begin{bmatrix} P_1 & P_2 \end{bmatrix}(t) \begin{bmatrix} \gamma^{-2} B_1 B_1^T & B_2 B_2^T \\ B_2 B_2^T & B_2 B_2^T \end{bmatrix}(t) \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}(t)$$

$$-\frac{dP_2(t)}{dt} = A^T(t) P_2(t) + P_2(t) A(t) + C_1^T(t) C_1(t) \quad (10)$$

$$-\begin{bmatrix} P_1 & P_2 \end{bmatrix}(t) \begin{bmatrix} 0 & \gamma^{-2} B_1 B_1^T \\ \gamma^{-2} B_1 B_1^T & B_2 B_2^T \end{bmatrix}(t) \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}(t)$$

Then:

i) the system $\dot{x}(t) = (A(t) - B_2(t) B_2^T(t) P_2(t)) x(t)$ is exponentially stable, so that $(A(t), B_2(t))$ is stabilizable

ii) If $(A(t) - \gamma^{-2} B_1(t) B_1^T(t) P_1(t), C_1(t))$ is detectable, then

   a) the system

   $$\dot{x}(t) = (A(t) - B_2(t) B_2^T(t) P_2(t) - \gamma^{-2} B_1(t) B_1^T(t) P_1(t)) x(t)$$

   is exponentially stable, and

   b) the strategies

   $$u^*(x,t) = -B_2^T(t) P_2(t) x(t) \quad (11)$$

   $$w^*(x,t) = -\gamma^{-2} B_1^T(t) P_1(t) x(t) \quad (12)$$

   lead to

   1) the closed-loop system when $u(t) = u^*(x,t)$ is exponentially stable such that $\|T_{zw}\|_\infty < \gamma$

   2) $J_2(w^*,u^*) \le J_2(w^*,u)$ provided that the system $\dot{x}(t) = (A(t) - \gamma^{-2} B_1(t) B_1^T(t) P_1(t)) x(t)$ is exponentially stable.

Conversely, suppose that $(A(t), B_2(t))$ is stabilizable and that there exist time-varying feedback strategies

$$\begin{aligned} u^*(x,t) &= K_2(t)x(t) \\ w^*(x,t) &= K_1(t)x(t) \end{aligned}$$

such that

(i) $\|T_{zw}\|_\infty < \gamma$ when $u(t) = u^*(x,t)$,

(ii) $J_2(w^*, u^*) \le J_2(w^*, u)$,

(iii) the system $\dot{x}(t) = (A(t) + B_2(t)K_2(t))x(t)$ is exponentially stable, and the system $\dot{x}(t) = (A(t) + B_1(t)K_1(t))x(t)$ is exponentially stable,

then there exist solutions $P_1(t) \le 0$ and $P_2(t) \ge 0$ satisfying (9) and (10).

Proof: See [22].

4.3 Static Output Feedback Case

Theorem 5: Consider (2) and suppose Assumptions A1) and A2) are satisfied. Suppose there exist a matrix $K(t)$ and symmetric matrices $Q_1(t) \ge 0$, $Q_2(t) \ge 0$ such that:

1) $R(t) = \gamma^2 I - D_k^T(t)D_k(t)$ is positive definite for all t

2)

$$\begin{aligned} -\dot{Q}_1(t) &= (A_k(t) + B_{1k}(t)R^{-1}(t)D_k^T(t)C_{1k}(t))^T Q_1(t) \\ &+ Q_1(t)(A_k(t) + B_{1k}(t)R^{-1}(t)D_k^T(t)C_{1k}(t)) \\ &+ Q_1(t)B_{1k}(t)R^{-1}(t)B_{1k}^T(t)Q_1(t) \\ &+ C_{1k}^T(t)(I + D_k(t)R^{-1}(t)D_k^T(t))C_{1k}(t) \end{aligned} \tag{13}$$

3)

$$\begin{aligned} &-\dot{Q}_2(t) = \\ &[A(t) + B_1(t)R^{-1}(t)(D_k^T(t)C_{1k}(t) + B_{1k}^T(t)Q_1(t)]^T Q_2(t) \\ &+Q_2(t)[A(t) + B_1(t)R^{-1}(t)(D_k^T(t)C_{1k}(t) + B_{1k}^T(t)Q_1(t))] \\ &-Q_2(t)B_2(t)B_2^T(t)Q_2(t) + C_1^T(t)C_1(t) \end{aligned} \tag{14}$$

4)

$$\begin{aligned} &K(t)C_2(t) + K(t)D_{21}(t)R^{-1}(t) \\ &\quad (D_k^T(t)C_{1k}(t) + B_{1k}^T(t)Q_1(t)) + B_2^T(t)Q_2(t) = 0 \end{aligned} \tag{15}$$

where

$$\begin{aligned} A_k(t) &= A(t) + B_2(t)K(t)C_2(t), \\ B_{1k}(t) &= B_1(t) + B_2(t)K(t)D_{21}(t), \\ C_{1k}(t) &= C_1(t) + D_{12}(t)K(t)C_2(t), \\ D_k(t) &= D_{12}(t)K(t)D_{21}(t). \end{aligned}$$

Then:

If the systems $\dot{x}(t) = A_k(t)x(t)$ and $\dot{x}(t) = (A(t) + B_1(t)R^{-1}(t)(D_k^T(t)C_{1k}(t) + B_{1k}^T(t)Q_1(t))x(t)$ are exponentially stable, the output feedback control $u^*(t) = K(t)y(t)$ and strategy $w^*(t) = R^{-1}(t)(D_k^T(t)C_{1k}(t) + B_{1k}^T(t)Q_1(t))x(t)$ will result in:

i) $\|T_{zw}\|_\infty < \gamma$ or $0 = J_1(w^*, u^*) \le J_1(w, u^*)$, and

ii) $J_2(w^*, u^*) \le J_2(w^*, u)$

Conversely, if there exist output feedback control $u^*(t) = K(t)y(t)$ and strategy $w^*(t) = K_1(t)x(t)$ such that:

a) The system $\dot{x}(t) = (A(t) + B_2(t)K(t)C_2(t))x(t)$ is exponentially stable;

b) $\|T_{zw}\|_\infty < \gamma$ or, $0 = J_1(w^*, u^*) \le J_1(w, u^*)$;

c) $J_2(w^*, u^*) \le J_2(w^*, u)$.

then $R(t) := \gamma^2 I - D_{21}^T(t)K^T(t)K(t)D_{21}(t) > 0$ for all t, and there exists a symmetric matrix $Q_1(t) \ge 0$ satisfying (13). In addition, $K_1(t) = R^{-1}(t)(D_k^T(t)C_{1k}(t) + B_{1k}^T(t)Q_1(t))$. Furthermore, if the system $\dot{x}(t) = (A(t) + B_1(t)R^{-1}(t)(D_k^T(t)C_{1k}(t) + B_{1k}^T(t)Q_1(t)))x(t)$ is exponentially stable, then there exists a symmetric matrix $Q_2(t) \ge 0$ satisfying (14) and (15).

Proof: See [22].

5 Conclusions

The report has totally achieved the aim of the original proposal of the project. This is summarized as follows.

In the report, state-space characterizations have been given for all exponentially stabilizing controllers and all H^∞ controllers for a given linear time-varying plant. State-space solutions have also been derived to the infinite horizon mixed H^2/H^∞ control problem for linear time-varying systems for both state feedback and static output feedback. The results obtained generalize some celebrated results in the literature.

Due to space limitation, all proofs are omitted. One can consult [23], [22] for more details.

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