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非線性控制理論與模擬-(子計畫三)
非線性系統低階 H^∞ 控制器設計 (III)
Reduced-order H^∞ controller design for nonlinear systems

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摘要

本文主要研究非線性動態系統低階 H^∞ 輸出回授控制器之設計，以耗散系統及微分對局理論觀點為架構，證明低階 H^∞ 控制器存在之條件，並求出其狀態空間表示式。其設計需解算兩組標準的 Hamilton-Jacobi 不等式及三個輔助方程式。

關鍵詞: H^∞ 控制，非線性動態系統，低階控制器設計

Abstract

Sufficient conditions are proposed for the existence of reduced-order (fixed-order) controllers solving the standard nonlinear H^∞ output feedback control problem. State-space formulas for such reduced-order H^∞ controllers are also derived in terms of the solutions of two Hamilton-Jacobi inequalities. The development uses only elementary concepts of dissipativity and differential game, thus the proofs given are simple and clear.

Keywords: H^∞ control, Nonlinear dynamical systems, Reduced-order controller design

1. Introduction

It has been shown that full-order H^∞ controllers can be constructed from two algebraic Riccati equations for linear systems or two Hamilton-Jacobi inequalities for nonlinear systems. The controllers thus obtained have a state dimension not less than that of the generalized plant [1], [5], [11], [13], [18], [20], [27], [28]. Since the generalized plant is built from the physical plant and some weighting functions that are used to reflect performance and robustness requirements, the order of generalized plant may be very high. In this case, the full-order controllers may be of limited use in practical applications.

Recently, a number of papers have appeared that deal with reduced-order (or fixed-order) H^∞ controller design for linear systems (see, e.g., [4], [6]-[10], [14]-[17], [19], [21], [22], [25], [26]). This project continues this line of research to study the reduced-order H^∞ controller design problem for nonlinear systems. In terms of the two standard Hamilton-Jacobi inequalities [11], we derive sufficient conditions for the existence of reduced-order (fixed-order) nonlinear H^∞ controllers, and give state-space formulas for such reduced-order nonlinear H^∞ controllers. The development uses only elementary concepts of dissipativity [24] and differential game [3], thus the proofs given are simple and clear.

2. Problem Formulation and Preliminaries

Consider a smooth (i.e. C^∞) nonlinear system described by the state equations

$$\dot{x} = f(x) + g_1(x)w + g_2(x)u \qquad (1a)$$
$$z = h_1(x) + k_{12}(x)u \qquad (1b)$$
$$y = h_2(x) + k_{21}(x)w \qquad (1c)$$

where x represents the state defined on a neighborhood of the origin in $I\!R^n$. Throughout, we assume that the origin is an equilibrium, i.e. $f(0) = 0$; without loss of generality, we assume also that $h_1(0) = 0$ and $h_2(0) = 0$. There are two inputs to the system: $u \in I\!R^{m_2}$ is the control input and $w \in I\!R^{m_1}$ represents a set of exogenous inputs which includes disturbances to be rejected and/or reference commands to be tracked. Eq.(1b) defines the controlled variable $z \in I\!R^{p_1}$ expressed as cost of the state x and the input u required to achieve the prescribed performance specifications. $y \in I\!R^{p_2}$ is the measured variable which is a function of the state x and the exogenous input w. In this report, we restrict ourselves to the consideration of systems satisfying the following standing assumption considered in, e.g., [11].

Assumption (A1): The matrices

$$R_1(x) := k_{12}^T(x)k_{12}(x)$$

and

$$R_2(x) := k_{21}(x)k_{21}^T(x)$$

are nonsingular for all x near $x = 0$.

Our aim in this report is to find a reduced-order (fixed-order) output feedback controller of the form

$$\begin{aligned} \dot{\xi} &= F(\xi) + G(\xi)y \\ u &= H(\xi) \end{aligned} \tag{2}$$

where $\xi \in I\!R^r$ $(r \leq n)$ is defined on a neighborhood of the origin, with $F(0) = 0$ and $H(0) = 0$, such that the resulting closed-loop system has a locally asymptotically stable equilibrium at the origin (x,ξ)=(0,0), and has L^2-gain $\leq \gamma$, or equivalently, such that there exists a neighborhood of the origin $(x, \xi) = (0,0)$ such that for all $T > 0$ and for each input $w(\cdot) \in L^2[0,T]$, the state trajectory of the closed-loop system starting from the initial state $(x(0), \xi(0)) = (0,0)$ remains in the neighborhood for all $t \in [0,T]$, and the response $z(\cdot)$ of the closed-loop system satisfies

$$\int_0^T \| z(t) \|^2 \, dt \leq \gamma^2 \int_0^T \| w(t) \|^2 \, dt.$$

For details, see [23].

We conclude this section by recalling from [11] the following results which provides an output feedback controller solving the problem in question.

Proposition 1 *Consider system* (1) *and suppose Assumption* (**A1**) *is satisfied. Suppose the following hypotheses hold.*

(H1) *There exists a smooth positive definite function $V(x)$, locally defined on a neighborhood of the origin in $I\!R^n$, such that the function*

$$\begin{aligned} Y_1(x) &= \frac{\partial V}{\partial x}(x)f(x) + h_1^T(x)h_1(x) + \gamma^2 w_*^T(x)w_*(x) \\ &\quad -u_*^T(x)R_1(x)u_*(x) \end{aligned} \tag{3}$$

is negative definite near $x = 0$, where

$$w_*(x) = \frac{1}{2\gamma^2}g_1^T(x)(\frac{\partial V}{\partial x})^T(x)$$

$$u_*(x) = -R_1^{-1}(x)(\frac{1}{2}g_2^T(x)(\frac{\partial V}{\partial x})^T(x) + k_{12}^T(x)h_1(x))$$

(H2) *There exists a smooth positive definite function $Q(x)$, locally defined on a neighborhood of the origin in $I\!R^n$, such that the function*

$$\begin{aligned} Y_2(x) &= \frac{\partial Q}{\partial x}(x)(\tilde{f}(x) - g_1(x)k_{21}^T(x)R_2^{-1}(x)\tilde{h}_2(x)) \\ &\quad +u_*^T(x)R_1(x)u_*(x) + \frac{1}{4\gamma^2}\frac{\partial Q}{\partial x}(x)g_1(x) \\ &\quad (I - k_{21}^T(x)R_2^{-1}(x)k_{21}(x))g_1^T(x)(\frac{\partial Q}{\partial x})^T(x) \\ &\quad -\gamma^2\tilde{h}_2^T(x)R_2^{-1}(x)\tilde{h}_2(x) \end{aligned}$$

is negative definite near $x = 0$ and its Hessian matrix is nonsingular at $x = 0$, where

$$\begin{aligned} \tilde{f}(x) &= f(x) + g_1(x)w_*(x) \\ \tilde{h}_2(x) &= h_2(x) + k_{21}(x)w_*(x) \end{aligned}$$

Then the nonlinear H^∞ output feedback control problem is solved by the output feedback

$$\begin{aligned} \dot{\hat{x}} &= \hat{f}(\hat{x}) + \hat{g}(\hat{x})y \\ u &= \hat{h}(\hat{x}) \end{aligned} \tag{4}$$

where $\hat{x} \in I\!R^n$ is defined on a neighborhood of the origin,

$$\begin{aligned} \hat{f}(\hat{x}) &= \tilde{f}(\hat{x}) + g_2(\hat{x})u_*(\hat{x}) - \hat{g}(\hat{x})\tilde{h}_2(\hat{x}) \\ \hat{h}(\hat{x}) &= u_*(\hat{x}) \end{aligned}$$

and $\hat{g}(\hat{x})$ satisfies

$$\frac{\partial Q}{\partial \hat{x}}(\hat{x})\hat{g}(\hat{x}) = (2\gamma^2\tilde{h}_2^T(\hat{x}) + \frac{\partial Q}{\partial \hat{x}}(\hat{x})g_1(\hat{x})k_{21}^T(\hat{x}))R_2^{-1}(\hat{x})$$

3. Main Results

In this section, we will propose a reduced-order controller of the form (2) that locally asymptotically stabilizes the resulting closed-loop system and renders its L^2-gain $\leq \gamma$. For this purpose, we first assume that there exists a smooth function $\psi : \mathbb{R}^r \to \mathbb{R}^n$ defined around the origin $\xi = 0$ in $\mathbb{R}^r$ with $\psi(0) = 0$ and $\text{rank}\frac{\partial \psi}{\partial \xi}(0) = r$. The rank condition implies that the restriction of ψ to some neighborhood Ω of $\xi = 0$ is an injection, provided by the injective mapping theorem [2]. We note that the inverse of the restriction of ψ to Ω (denoted by $\phi := (\psi|_\Omega)^{-1}$) is continuous around the origin $x = 0$ in $\mathbb{R}^n$ but need not be differentiable. For our purposes, we shall assume ϕ to be at least continuously differentiable. Then we make a change of variables

$$\hat{\xi} = -\phi(x) + \xi \tag{5}$$

where $\hat{\xi} \in \mathbb{R}^r$ and $\xi \in \mathbb{R}^r$ are defined on a neighborhood of the origin. In terms of these variables the resulting closed-loop system is

$$\begin{array}{rcl} \dot{x}_e & = & f_e(x_e) + g_e(x_e)w \\ z & = & h_e(x_e) \end{array} \tag{6}$$

where $x_e = col(x, \hat{\xi})$,

$f_e(x_e) =$

$$\begin{bmatrix} f(x)+g_2(x)H(\hat{\xi}+\phi(x)) \\ -\frac{\partial \phi}{\partial x}(x)f(x)-\frac{\partial \phi}{\partial x}H(\hat{\xi}+\phi(x))+F(\hat{\xi}+\phi(x))+G(\hat{\xi}+\phi(x))h_2(x) \end{bmatrix},$$

$$g_e(x_e) = \begin{bmatrix} g_1(x) \\ -\frac{\partial \phi}{\partial x}(x)g_1(x) + G(\hat{\xi} + \phi(x))k_{21}(x) \end{bmatrix},$$

and

$$h_e(x_e) = h_1(x) + k_{12}(x)H(\hat{\xi} + \phi(x)).$$

We are now in the position to state our main result. The proof can be found in [29].

Theorem 2 *Suppose that Assumption* **(A1)** *is satisfied and that Hypotheses* **(H1)** *and* **(H2)** *of Proposition 1 hold. Suppose that there exists a smooth function $\psi : \mathbb{R}^r \to \mathbb{R}^n$, locally defined on a neighborhood of the origin $\xi = 0$ in $\mathbb{R}^r$, with $\psi(0) = 0$ and $\text{rank}\frac{\partial \psi}{\partial \xi}(0) = r$ such that the inverse of the restriction of ψ to some suitable neighborhood of $\xi = 0$, denoted by ϕ, is continuously differentiable and satisfies $\frac{\partial \phi}{\partial x}(0)(\frac{\partial \phi}{\partial x})^T(0) = I$. Suppose also that there exists a smooth positive definite function U, locally defined on a neighborhood of the origin $\xi = 0$ in $\mathbb{R}^r$, which satisfies $\frac{\partial^2 U}{\partial \xi^2}(0)\frac{\partial \phi}{\partial x}(0) = \frac{\partial \phi}{\partial x}(0)\frac{\partial^2 Q}{\partial x^2}(0)$. Then, if F, G, and H satisfy*

$$F(\phi(x)) = \frac{\partial \phi}{\partial x}(x)\hat{f}(x), \tag{7}$$

$$G(\phi(x)) = \frac{\partial \phi}{\partial x}(x)\hat{g}(x), \tag{8}$$

and

$$H(\phi(x)) = \hat{h}(x), \tag{9}$$

the r-th order controller (2) *locally asymptotically stabilizes the resulting closed-loop system* (6) *and renders its L^2-gain $\leq \gamma$.*

Remark: It has been shown in Theorem 3 that the achievement of closed-loop asymptotic stability is implied by the fulfilment of the negative definiteness of $Y_1(x)$ and $Y_2(x)$. If $Y_1(x)$ is just negative semidefinite near $x = 0$, then closed-loop asymptotic stability can still be achieved if the equilibrium $\xi = 0$ of the system $\dot{\xi} = F(\xi)$ is locally asymptotically stable (i.e., the controller (2) itself is internally stable) and system (1) satisfies the following standard assumption [11] usually considered in the study of the full-order H^∞ controller design problem. See [29] for more discussions.

Assumption (A2): Any bounded trajectory $x(t)$ of the system

$$\dot{x}(t) = f(x(t)) + g_2(x(t))u(t)$$

satisfying

$$h_1(x(t)) + k_{12}(x(t))u(t) = 0$$

for all $t \geq 0$, is such that $\lim_{t \to \infty} x(t) = 0$.

For completeness, we give the linear version of Theorem 3 in the following statement. See [29] for proof.

Proposition 3 *Suppose that the system is linear and described by*

$$\begin{array}{rcl} \dot{x} & = & Ax + B_1 w + B_2 u \\ z & = & C_1 x + D_{12} u \\ y & = & C_2 x + D_{21} w. \end{array} \tag{10}$$

Suppose that the matrices $R_1 := D_{12}^T D_{12}$ and $R_2 := D_{21} D_{21}^T$ are nonsingular. Suppose also that the following hypotheses hold.

(L1) *There exists a symmetric positive definite matrix X satisfying the Riccati inequality*

$$(A-B_2R_1^{-1}D_{12}^TC_1)^TX+X(A-B_2R_1^{-1}D_{12}^TC_1)$$
$$+C_1^T(I-D_{12}R_1^{-1}D_{12}^T)C_1+X(\frac{1}{\gamma^2}B_1B_1^T-B_2R_1^{-1}B_2^T)X<0$$

(L2) *There exists a symmetric positive definite matrix S satisfying the Riccati inequality*

$$(\tilde{A}-B_1D_{21}^TR_2^{-1}\tilde{C}_2)^TS+S(\tilde{A}-B_1D_{21}^TR_2^{-1}\tilde{C}_2)$$
$$+\tilde{F}^TR_1\tilde{F}-\gamma^2\tilde{C}_2^TR_2^{-1}\tilde{C}_2+\frac{1}{\gamma^2}SB_1(I-D_{21}^TR_2^{-1}D_{21})B_1^TS<0,$$

where

$$\begin{aligned}\tilde{A} &= A+\frac{1}{\gamma^2}B_1B_1^TX\\ \tilde{C}_2 &= C_2+\frac{1}{\gamma^2}D_{21}B_1^TX\end{aligned}$$

and

$$\tilde{F}=-R_1^{-1}(B_2^TX+D_{21}^TC_1)$$

Furthermore, suppose that V is an $n\times r$ matrix which satisfies $V^TV=I$. Then the r-th order controller

$$\begin{aligned}\dot{\xi} &= F\xi+Gy\\ u &= H\xi\end{aligned}$$

internally stabilizes (10) *and makes the H^∞ norm of the closed-loop transfer matrix from w to z less than or equal to γ if F, G and H satisfy*

$$FV^T=V^T(\tilde{A}+B_2\tilde{F}-\tilde{G}\tilde{C}_2) \qquad (11)$$

$$G=V^T\tilde{G} \qquad (12)$$

and

$$HV^T=\tilde{F} \qquad (13)$$

with

$$\tilde{G}=(\gamma^2S^{-1}\tilde{C}_2^T+B_1D_{21}^T)R_2^{-1}.$$

4. Conclusions

The report has totally achieved the aim of the original proposal of the project. This is summarized as follows.

In the report, a method has been proposed for designing reduced-order H^∞ controllers of nonlinear systems. It has been shown that reduced-order nonlinear H^∞ controllers can be constructed from the solutions of two Hamilton-Jacobi inequalities and three auxiliary equations.

Due to space limitation, all proofs are omitted. One can consult [29] for more details.

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