

國立臺灣海洋大學一〇二學年度研究所碩士班暨碩士在職專班招生考試試題

考試科目： 計算機數學（含線性代數、離散數學）

系所名稱： 資訊工程學系碩士班不分組

1.答案以橫式由左至右書寫。2.請依題號順序作答。

1. (10%) Find all solutions of the given linear system.

$$x_1 + 2x_2 - 3x_3 - 4x_4 = 2$$

$$2x_1 + 4x_2 - 5x_3 - 7x_4 = 7$$

$$-3x_1 - 6x_2 + 11x_3 + 14x_4 = 0$$

2. (10%) Find the projection of $[1, 0, -1, 1]$ on the subspace of W , where W is spanned by the linear combinations of vectors $[1, 0, 0, 0]$, $[0, 1, 1, 0]$ and $[0, 0, 1, 1]$ in \mathbb{R}^4 .

3. (10%) Given the following matrix A , compute $A^{101} = ?$

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 3 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4. (10%)

Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the linear transformation defined by

$T([x, y, z]) = [x + y + z, x + y, z]$. Find the matrix representation A of T relative to B , where $B = ([1, 1, 0], [1, 0, 1], [0, 1, 1])$.

5. (10%) Let A be an $m \times n$ matrix with $\text{rank}(A) = r$, prove that $\text{rank}(A^T A) = r$, where A^T is the transpose of A .

6. Are there infinitely many primes? why? (15%)

7. $a_n = 8a_{n-1} + 10^{n-1}$, $a_1 = 9$

Use generating functions to solve it (20%)

8. $\binom{n+1}{r+1} = \sum_{j=r}^n \binom{j}{r}$ why? (15%)