考試科目： 控制系統（含線性系統理論）

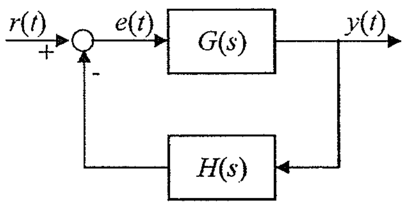
系所名稱： 電機工程學系碩士班控制組

1.答案以橫式由左至右書寫。2.請依題號順序作答。

1. (12%) For the transfer function given below, please find the constraints on K_1 and K_2 such that the transfer function will have all its poles in the open right half plane.

$$H(s)=\frac{s^2+3s+2}{s^4-2s^3+K_1s^2-4s+K_2}$$

2. (18%) Consider the following feedback system:



where $G(s)=\dfrac{K}{(s+2)^3(s+5)}$ and $H(s)=1$.

(1) Please find the range of K such that the real parts of the closed-loop poles are less than -1.

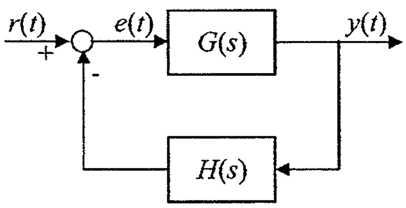
(2) If $K=10$ and the input is a unit step function, i.e.,

$$r(t)=u_s(t)=\begin{cases}0, \text{if } t<0\\ 1, \text{if } t\geq 0,\end{cases}$$

please find the steady state error $e_{ss}=\lim_{t\to\infty}e(t)$.

(3) If $K=-10$ and the input is also a unit step function, please find $y_{ss}=\lim_{t\to\infty}y(t)$.

3. (16%) Consider the following feedback system:



where $G(s)=\dfrac{s+\alpha}{s(s+3)(s+6)}$ and $H(s)=K$.

(1) Find the values of α and K that will yield a pair of closed-loop poles at $-1\pm j100$, where $j=\sqrt{-1}$.

(2) In this case, please find the remaining closed-loop pole.

4. (8%) Consider a linear control system:

$$\dot{x} = Ax + Bu .$$

Suppose that λ is an eigenvalue of A and v is a corresponding left eigenvector (*i.e.*, $vA = \lambda v$, v is a nonzero row vector). Suppose that $vB = 0$. Please prove that λ will be a closed-loop pole for any possible state feedback controller $u = Fx$.

5. (10%) Consider a linear system

$$\dot{x} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u .$$

Please find a state feedback controller $u = Fx$ such that the closed-loop poles are $\{-2 + j2, -2 - j2, -3\}$.

6. (18%) Consider a linear system

$$\dot{x} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & -4 \\ 0 & -1 & 2 \end{bmatrix} x$$

$$y = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} x$$

(1) Is this system observable? Please justify your answer (*i.e.*, you must provide a reason for your answer).

(2) If it is not observable, please find the unobservable eigenvalue(s).

(3) Is it detectable? Please justify your answer.

7. (18%) Consider the following linear control system:

$$\dot{x}(t) = Ax(t) + Bu; x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$y(t) = Cx(t)$$

where

$$A = \begin{bmatrix} -1 & -1 \\ 2 & -4 \end{bmatrix}, B = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

(1) Please find e^{At}.

(2) Find the zero input response $y(t)$.

(3) Let input $u(t) = \begin{cases} e^{-2t}, \text{if } t \geq 0 \\ 0, \quad \text{if } t < 0. \end{cases}$. Please find $x(t)$ for $t \geq 0$.