

國立臺灣海洋大學一〇二學年度研究所碩士班暨碩士在職專班招生考試試題

考試科目： 線性代數

系所名稱： 電機工程學系碩士班控制組

1.答案以橫式由左至右書寫。2.請依題號順序作答。

- (1) (15%) Find the singular value decomposition of the matrix

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}.$$

- (2) (15%) Let  $A, B$  be  $n \times n$  matrices. Prove that  $AB$  has the same eigenvalues as  $BA$ .
- (3) (15%) Given

$$A = \begin{pmatrix} 0 & i & 0 \\ i & 1 & i \\ 0 & i & 0 \end{pmatrix}.$$

Find a  $3 \times 3$  unitary matrix  $U$  such that  $U^*AU$  is diagonal. Here  $i = \sqrt{-1}$ , and  $U^*$  stands for the conjugate transpose of  $U$ .

- (4) (25%) Let

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

- (a) Find the characteristic polynomial of  $A$ .
- (b) Find the Jordan canonical form of  $A$ .
- (c) Find the minimal polynomial of  $A$ .
- (5) (15%) Find a basis and the dimension of the subspace  $W$  of  $\mathbb{R}^{2 \times 2}$  spanned by  $A = \begin{pmatrix} 1 & -5 \\ -4 & 2 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ -1 & 5 \end{pmatrix}, C = \begin{pmatrix} 2 & -4 \\ -5 & 7 \end{pmatrix}, D = \begin{pmatrix} 1 & -7 \\ -5 & 1 \end{pmatrix}$ .
- (6) (15%) Let  $A$  be a  $2 \times 2$  real matrix such that only  $A$  is similar to itself. Show that  $A$  is a scalar matrix, that is, that  $A = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}$  for some  $a \in \mathbb{R}$ .