



考試科目： 工程數學

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1. 答案以橫式由左至右書寫。2. 請依題號順序作答。

1. (a). Plot $\cos(x)$ versus x . (請畫 $\cos(x)$ 對 x 作圖) (5 %)

(b) Write down the relationship among e, π and i using $e^{\pi i} = ?$.

where $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots, \pi = 3.14159 \dots$ and $i^2 = -1$. (5 %)

(c). Determine $e^{ix} - \cos(x) - i \sin(x) = ?$ (5 %)

(d). If the three roots for $z^3 = 1$ are z_1, z_2, z_3 , please find $z_1 + z_2 + z_3 = ?$ (5 %)

(e). Write down the Cauchy integral formula. (5 %)

2. Given a matrix

$$A = \begin{bmatrix} 0 & -3 & 2 \\ 3 & 0 & -1 \\ -2 & 1 & 0 \end{bmatrix}$$

Find the rank and nullity of matrix A (5 %)

If $Ay = \omega \times y$ for any y vector, find the vector ω (5 %)

Find the determinant of A. (5 %) Is the matrix A singular? (5 %) Is zero the eigenvalue of matrix A? (5 %)

3.(a) Find the two complementary solutions $y_1(t)$ and $y_2(t)$ of $y''(t) + y(t) = 0$ (5%)

(b) Find the Wronkian of the two complementary solutions. (5%)

(c) Find the particular solution of $y''(t) + y(t) = \cos(\Omega t)$ (5%)

(d) What is the particular solution when $\Omega \rightarrow 1$ (5%), what is beating? (5%)

(e) Using variation of parameters, $y_p(t) = y_1(t)u_1(t) + y_2(t)u_2(t)$ to solve the particular solution for

$$y''(t) + y(t) = \cos(t) \quad (5 \%)$$

4. Consider $f(x) = x + \pi$, $-\pi < x < \pi$ (period 2π)

(a) Determine whether the function f is even, odd, or neither (5 %)

(b) Find the Fourier series of f on the given interval $(-\pi, \pi)$ (10 %)

(c) Find the values that the series will converge at $x = -\pi, 0, \pi/2, \pi$ (5 %) (converge in the mean)

(d) Use the result of (b) to find $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = ?$ (5 %)