國立臺灣海洋大學100學年度轉學生入學招生考試試題

考試科目： 微積分

系所名稱： 日商船二、日輪機能應二、日輪機動力二、日環漁二、日環資二、日電機二 、日河工二

1.答案以橫式由左至右書寫。2.請依題號順序作答。

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1. 敘述微積分基本定理。(10 分)
2. 計算 (每小題 5 分)

(i) $\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{x}$

(ii) $y=\begin{cases}x^2\sin\frac{1}{x}, & x\neq 0\\ 0, & x=0\end{cases}$，求$\frac{dy}{dx}$。

(iii) $xy^2+x^2y=2$，求$\frac{dy}{dx}$。

(iv) $\begin{cases}x=t-\sin 2t\\ y=1-\cos t^2\end{cases}$，求 dy/dx。

3. 作 $y=\frac{x^3}{x^2-4}$ 的圖形。(10 分)
4. 求曲面 $z=x^2+y^2$與平面 $z=x+2y-1$ 所交曲線在(1,1,2)的切線方程式。(5 分)
5. 設 f(x)為 由[0,1]區間到 [0,1]的連續函數，證明在 [0,1] 內必有一數 c 滿足 $f(c)=c$。(5 分)

6. (12%) Evaluate the following integrals.

$$(a)\ \int \left(x^3 - \frac{2}{x^3} + \frac{3}{\sqrt[3]{x}}\right) dx \qquad (b)\ \int \frac{1}{1+e^{-x}}\, dx \qquad (c)\ \int_1^e x^3 \ln x\, dx$$

7. (8%) Evaluate the improper integrals and determine whether it converges or diverges. $(a)\ \int_{-\infty}^{1} \frac{1}{(3-2x)^{3/2}}\, dx \qquad (b)\ \int_1^e \frac{1}{x \ln x}\, dx.$

8. (10%) Evaluate the following double integrals:

$(a)\ \int_0^2 \int_{-x}^{3x} (e^x + 2xy)\, dy\, dx$

$(b)\ \int_0^6 \int_{\frac{\pi}{3}}^{2} e^{y^2}\, dy\, dx$ (hint: changing the order of integration).

9. (8%) (a) Find the *convergence set* (or interval) for the power series $\sum_{n=1}^{\infty} \frac{(3x+1)^n}{n \cdot 2^n}$.

(b) Use the geometric series of $(1-x)^{-1} = 1 + x + x^2 + \cdots$ and the fact that $\tan^{-1} x = \int_0^x \frac{1}{1+t^2}\, dt$ to find the first four nonzero terms in the *Maclaurin series* for $\tan^{-1} x$.

10. (12%) The following figure shows the graph of a function f that has a continuous third derivative. The dashed lines are tangent to the graph of $y = f(x)$ at the points $(-1, 2)$ and $(3, 1)$. Based on what is shown, determine whether the following integrals are positive, negative, or zero, and also give some reasons. (看圖算出下列積分是正, 負, 或零, 並且說明理由).

$$(a)\ \int_{-1}^{3} f(x)\, dx \qquad (b)\ \int_{-1}^{3} f'(x)\, dx \qquad (c)\ \int_{-1}^{3} f''(x)\, dx \qquad (d)\ \int_{-1}^{3} f'''(x)\, dx$$