



國立臺灣海洋大學 100 學年度轉學生入學招生考試試題

考試科目：微積分

\*可使用計算機

系所名稱：日資工二、日通訊二

1. 答案以橫式由左至右書寫。2. 請依題號順序作答。

1. 敘述以下：

(i)  $f(x)$  在  $x = a$  的導數的定義。(5 分)

(ii) 均值定理。(The Mean Value Theorem)(5 分)

2. 計算：

(i)  $\lim_{x \rightarrow -\infty} \left(1 - \frac{2}{x}\right)^x$  (5 分)

(ii)  $\frac{d}{dx} \left[ \int_x^{x^2} \sqrt{1+t^2} dt \right]$  (5 分)

(iii)  $\begin{cases} x = 2t - t^2 \\ y = 3t - t^3 \end{cases}$ , 求  $\frac{dy}{dx}$  (5 分)

(iv)  $f(x, y) = \tan^{-1} \frac{y}{x}$ , 求  $\nabla f(x, y)$  (5 分)

3. 作  $y = \frac{x^2+1}{x}$  的圖形。(10 分)

4. 證明方程式  $x^{101} + x^{51} + x - 1 = 0$  恰有一實根。(5 分)

5. 設  $0 < x < 1$  求  $\sum_{n=0}^{\infty} nx^n$  之和。(5 分)

6. (15%) Evaluate the following integrals.

$$(a) \int e^{2x} \sin 3x \, dx \quad (b) \int_{-4}^{-\frac{4}{\sqrt{3}}} \frac{\sqrt{t^2 - 4}}{t^3} \, dt \quad (c) \int_0^{\pi/2} \frac{\cos x}{1 + \sin x} \, dx$$

7. (10%) Evaluate the improper integrals and determine whether it converges or diverges.

$$(a) \int_0^{3/2} \frac{dx}{(3 - 2x)^{4/3}} \quad (b) \int_1^{\infty} x e^{-\frac{x^2}{2}} \, dx.$$

8. (10%) Evaluate the following double integrals.

$$(a) \iint_R (x^2 + 2xy) \, dA, \quad \text{where } R \text{ is the region between } y = x^2 \text{ and } y = \sqrt{x}.$$

$$(b) \int_0^4 \int_{x/2}^2 e^{-y^2} \, dy \, dx \quad (\text{hint: changing the order of integration}).$$

9. (15%) (a) Find the *convergence set* (or interval) for the power series  $\sum_{n=1}^{\infty} \frac{(2x+1)^n}{n \cdot 3^n}$ .

(b) Let the series  $\sum_{k=1}^{\infty} \frac{k}{e^{k^2}}$  converges to its sum,  $S$  say, and we now try to use the  $n$ th partial sum,  $S_n = \sum_{k=1}^n \frac{k}{e^{k^2}}$ , to approximate  $S$ . How large  $n$  must be so that the error no

more than the *required tolerance*  $\delta = 0.000001$  (所要求之誤差容忍度). (hint: Applying the *integral test* to get an error bound first and then using the values  $\ln 2 \approx 0.6931$  and  $\ln 5 \approx 1.6094$  and calculator).

(c) Use the binomial series of  $(1+x)^{-1/3}$  find the *Maclaurin series* for the function

$$f(x) = \int_0^x \frac{1}{\sqrt[3]{1-t^2}} \, dt \quad (\text{At least give first four nonzero terms}).$$