

國立臺灣海洋大學九十九學年度研究所碩士班暨碩士在職專班入學考試試題

考試科目：數學(含微積分及線性代數)

系所名稱：商船學系碩士班甲組、乙組

1. 答案以橫式由左至右書寫。2. 請依題號順序作答。

1. (16%) Find $\frac{dy}{dx}$ of the following functions

(a) $y = x^{\sqrt{x}}$, (b) $y = \frac{(3x+2)^3}{(2x-1)^4(x-4)^5}$, (c) $y = \cos xy$, (d) $y = \int_{e^{2x}}^{e^{x^2}} \ln t^2 dt$.

2. (16%) Evaluate

(a) $\int \tan x dx$, (b) $\int \ln x dx$, (c) $\int \frac{1}{x^2-4} dx$, (d) $\int_0^{\infty} \frac{1}{e^x + e^{-x}} dx$.

3. (8%) If $f(x) = \begin{cases} x^2 & x \leq 2 \\ ax + b & x > 2 \end{cases}$, and $f'(2)$ exists, find a and b .

4. (6%) If $f\left(\frac{1+x}{1-x}\right) = x$, find $f'(x)$.

5. (6%) Find the dimensions of a right circular cylinder of maximum volume that can be inscribed in a sphere of radius 10cm. What is the maximum volume?

6. (6%) Find the length of the curve $y = \frac{4\sqrt{2}}{3}x^{3/2} - 1$, with $0 \leq x \leq 1$.

7. The nonhomogeneous linear system $A\mathbf{X} = \mathbf{B}$, where $A = \begin{pmatrix} -1 & -2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 2 & 1 \\ 1 & 2 & -3 & -7 & -2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix}$

(a) (6%) Find $\text{rank}(A)$ and $\text{rank}([A|\mathbf{B}])$.

(b) (8%) Find the general solution of $A\mathbf{X} = \mathbf{0}$.

(c) (8%) Find the general solution of $A\mathbf{X} = \mathbf{B}$.

8. Let $A = \begin{pmatrix} 0 & -2 \\ 1 & 3 \end{pmatrix}$

(a) (6%) Find the eigenvalues of the matrix and, for each eigenvalue, a corresponding eigenvector.

(b) (6%) Is the matrix diagonalizable?

(c) (4%) Evaluate e^A .

(d) (4%) If $P(x) = x^3 + 3x^2 - 3x - 1$, find $P(A)$.