

國立臺灣海洋大學九十九學年度研究所碩士班暨碩士在職專班入學考試試題

考試科目：工程數學

系所名稱：輪機工程學系碩士班不分組

1. 答案以橫式由左至右書寫。2. 請依題號順序作答。

1. (10%) Solve the given homogeneous equation by using an appropriate substitution.

$$-ydx + (x + \sqrt{xy})dy = 0$$

2. (12%) Solve $x^2 y'' - 3xy' + 3y = 2x^4 e^x$

3. (12%) Use the Laplace Transform to solve the given initial-value problem.

$$y'' - y' = e^t \cos t \quad y(0) = 0 \quad y'(0) = 0$$

4. (12%) Determine whether the given matrix A is diagonalizable. If so, find the matrix P that diagonalizes A and the diagonal matrix D such that $D = P^{-1}AP$

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & -2 \\ -5 & 3 & 8 \end{bmatrix}$$

5. (10%) Find the curl and divergence of the given vector field.

$$\vec{F}(x, y, z) = xye^x \vec{i} + x^3 yze^z \vec{j} + xy^2 e^y \vec{k}$$

6. (10%) Use Green's theorem to evaluate the given line integral

$$\oint_C (x + y^2)dx + (2x^2 - y)dy \quad \text{where } C \text{ is the boundary of the region}$$

determined by the graphs of $y = x^2$, $y = 4$

7. (12%) Use the divergence theorem to find the outward flux $\iint_S (\vec{F} \cdot \vec{n})ds$ of the

given vector field \vec{F} , $\vec{F} = x^2 \vec{i} + 2yz \vec{j} + 4z^3 \vec{k}$; D the region bounded by

the parallelepiped defined by $0 \leq x \leq 1$, $0 \leq y \leq 2$, $0 \leq z \leq 3$

8. (12%) Expand the given function in an appropriate cosine or sine series

$$f(x) = \begin{cases} x+1 & -1 < x < 0 \\ x-1 & 0 \leq x < 1 \end{cases}$$

9. (10%) Find all values of the given quantity

$$(-i)^{4i}$$