1. (a) $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 2 \end{pmatrix}, B = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$. First compute AB, then BA. (8%)

(b) Assume A and B are some 2×2 matrices such that $AB-BA = \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}$ Find $A^tB^t - B^tA^t$. (5%)

2. (a) Using the formula for the inverse in terms of the adjoint, find the inverse of the matrix

$A = \begin{pmatrix} \cos\theta & 1 & -\sin\theta \\ 0 & 2 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}$ (10%)

(b) Using Cramer's rule, find the solution of the system. (10%)

$\begin{cases} 2X_1 + X_2 & = 1 \\ X_1 + 2X_2 + X_3 = 0 \\ X_2 + 2X_3 = 0 \end{cases}$

(the determinant of the matrix associated to the system is 4)

3. 請計算 $\dfrac{[\frac{3}{2}\sqrt{3} + \frac{3}{2}i]^6}{[\sqrt{\frac{5}{2}} + i\sqrt{\frac{5}{2}}]^3}$。(6%)

4. 令 z_k 是方程式 $(z+1)^7 + z^7 = 0$ 的根，請計算 $z_k =$? (7%)

5. $f(z) = ((z+2)^3 + z^3)^4$，試計算 $\dfrac{df}{dz}$。(5%)

6. 請計算 $\displaystyle\int_C \dfrac{\sin\pi(z+1) + \cos\pi z}{(z-1)(z-2)} dz$，其中 C 為 $|z| = 4$ 的圓。(5%)

7. 求下列各函數的留數 (a) $\dfrac{z^2+z}{(z-1)^2(z^2+4)}$; (b) $\cot z$。(10%)

8. It is known that y is a function of x with $-5 < x < 5$. If $\dfrac{dy(x)}{dx} = -x / y(x)$ and $y(4) = -3$, please solve $y(x)$ and express $y(x)$ in terms of x. (10%)

9. Solve **the general solution** for the following equation: $\dfrac{d}{dx}(\dfrac{d}{dx}y(x)) - 4\dfrac{dy(x)}{dx} = 10e^{-2x} - 3y$.

(10%)

10. Solve the (i)divergence, (ii)gradience and (iii)Laplacian of the following vector function:

$x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$. (9%)

11. Prove that $\dfrac{d}{dx}a^x = \ln(a) \cdot a^x$ through the help of chain rule:

$\dfrac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$. (5%)