

國立臺灣海洋大學 101 學年度研究所碩士班暨碩士在職專班入學考試試題

考試科目：工程數學

系所名稱：輪機工程學系碩士班不分組

1. 答案以橫式由左至右書寫。2. 請依題號順序作答。

每題 10 分，共 10 題，總分 100 分

1. Solve $y'' + y = 4\cos x$

2. Use the Laplace Transform to solve the given initial-value problem

$$y'' - 6y' + 9y = t^2 e^{3t} \quad \text{subject to } y(0) = 2 \quad y'(0) = 17$$

3. Find the inverse of $A = \begin{bmatrix} 2 & 2 & 0 \\ -2 & 1 & 1 \\ 3 & 0 & 1 \end{bmatrix}$

4. Find the eigenvalues and eigenvectors of the given matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & -7 \end{bmatrix}$$

5. Determine whether the given matrix A is diagonalizable. If so, find the matrix P that diagonalizes A and the diagonal matrix D such that $D = P^{-1}AP$

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & -2 \\ -5 & 3 & 8 \end{bmatrix}$$

6. Find the curl and divergence of the given vector field

$$\vec{F}(x, y, z) = x^2 \sin yz \vec{i} + z \cos xz^3 \vec{j} + ye^{xy} \vec{k}$$

7. Use Green's Theorem to evaluate the given line integral

$$\oint_C 2xy dx + 3xy^2 dy, \quad \text{where } C \text{ is the triangle with vertices } (1,2) (2,2) (2,4)$$

8. Use Stokes' Theorem to evaluate $\oint_C \vec{F} \cdot d\vec{r}$. Assume C is oriented counterclockwise as viewed from above. $\vec{F} = (x+2z)\vec{i} + (3x+y)\vec{j} + (2y-z)\vec{k}$ C the curve of intersection of the plane $x+2y+z=4$ with the coordinate planes.

9. Find the Fourier Series of f on the given interval

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 \leq x < \pi \end{cases}$$

10. Solve $(e^{2y} - y \cos xy)dx + (2xe^{2y} - x \cos xy + 2y)dy = 0$