國立臺灣海洋大學 100 學年度博士班招生考試試題

考試科目： 工程數學

系所名稱： 系統工程暨造船學系博士班　　※可使用計算器

1.答案以橫式由左至右書寫。2.請依題號順序作答。

1. [20%]Consider a vector $\boldsymbol{a}=\left(a_x,a_y\right)$ and a matrix $\boldsymbol{R}=\begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}$
   (a) Calculate the vector $\boldsymbol{b}=\boldsymbol{R}\cdot\boldsymbol{a}$
   (b) (vector $\boldsymbol{b}$ is obtained from (a)) Calculate the inner product $\boldsymbol{c}=\boldsymbol{b}\cdot\boldsymbol{a}$ and cross product $\boldsymbol{d}=\boldsymbol{b}\times\boldsymbol{a}$ and the angle between vector $\boldsymbol{a}$ and $\boldsymbol{b}$
   (c) Explain the meaning of $\boldsymbol{b}=\boldsymbol{R}\cdot\boldsymbol{a}$

2. [20%] Define a surface function $f\left(x,y,z\right)=ax+by+cz+d=0$ in three-dimensional space. Find the coefficients *a*, *b*, *c* and *d* if the surface pass points $\left(1,2,3\right)$, $\left(8,6,4\right)$ and $\left(1,3,5\right)$

3. [20%]Given a vector $\boldsymbol{u}$ in space *S*，and an orthonormal set $\left\{\hat{e}_1,\ldots,\hat{e}_N\right\}$ in *S*, where *N* is smaller than the dimension of the space *S*. the best approximation of $\boldsymbol{u}$ is defined as follows

$$\boldsymbol{u}^h\approx\sum_{i=1}^{N}c_j\hat{\boldsymbol{e}}_j=\boldsymbol{V}\boldsymbol{c}\qquad \begin{matrix}V=\left[\hat{e}_1,\ldots,\hat{e}_N\right]\\ \boldsymbol{c}=\left[c_1,c_2,\ldots,c_N\right]^T\end{matrix}\tag{1}$$

To solve the coefficient vector $\boldsymbol{c}$ in equation (1), the least square minimization is introduced, which defines the squared norm of the error as a functional

$$\Pi\left(\boldsymbol{c}\right)=\left\|\boldsymbol{E}\right\|^2=\left\|\boldsymbol{u}-\boldsymbol{u}^h\right\|^2\tag{2}$$

By taking the variation of the functional, please derive/show the following
(a) The explicit expression of the coefficient vector $\boldsymbol{c}$
(b) The error of the approximation $\boldsymbol{E}$ is orthogonal to the approximation $\boldsymbol{u}^h$

4. [20%]Consider the following equation

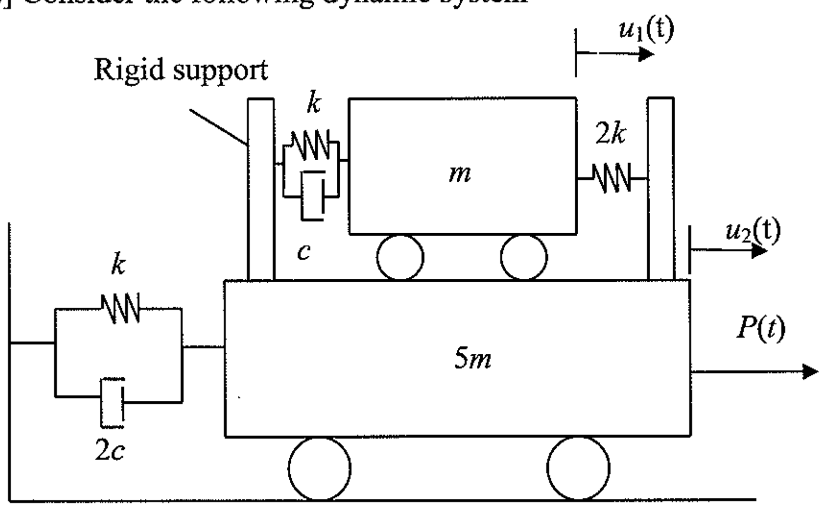
$$\frac{\partial u}{\partial t}+a\frac{\partial u}{\partial x}=0 \qquad (3)$$

The following Lax-Wendroff scheme is proposed to solve equation (3) numerically

$$u_i^{n+1}=u_i^n-\frac{\lambda}{2}\left(u_{i+1}^n-u_{i-1}^n\right)+\frac{\lambda^2}{2}\left(u_{i+1}^n-2u_i^n+u_{i-1}^n\right), \qquad \lambda=a\frac{\Delta t}{\Delta x} \qquad (4)$$

where n and i denote the discrete time and space respectively. Prove equation (4) is 2^{nd} order accurate both in space x and in time t.

5. [20%] Consider the following dynamic system



Where k is the spring constant, c is the damping coefficient, m is the mass and $P(t)$ is the applied external force.

(a) Construct the equation of motion for the system

(b) Assume the damping coefficient c is zero, find the modal frequencies and its associated mode shapes