國立臺灣海洋大學一００學年度研究所碩士班暨碩士在職專班入學考試試題

考試科目： 統計學

系所名稱： 應用經濟研究所碩士班不分組

※可使用計算器

1.答案以橫式由左至右書寫。2.請依題號順序作答。

說明：(1) 本試題共 5 大題，所有題目必須全部作答，總分 100 分。
(2) 如果您對題目有問題，請立即提出或自行做假設(make your own assumption(s)，並在答案紙上作說明。

1. (25 分) 設隨機變數 $x\sim(\mu_x,\sigma_x^2)$，自 x 中隨機抽取 n>30 為一組樣本$(x_1,x_2,....,x_n)$，令 $\bar{x}=\frac{\sum_{i=1}^{n} xi}{n}$，則

(a) (5 分) 檢定$H_0:\mu\le 3$；$H_1:\mu>3$，若臨界值向右移，則型 I 誤差α=Prob(Reject $H_0:\mu\le 3|H_0:\mu\le 3$) 變大，而型 II 誤差$\beta$=Prob(Accept $H_0:\mu\le 3|H_1:\mu>3$) 變小。(不論「是」或「否」均請說理由，否則不予給分)

(b) (10 分) 今欲將$\bar{x}$的標準誤加倍，則抽樣樣本大小應如何調整？

(c) (10 分) 若欲估計μ^2，則是否$\bar{x}^2$為μ^2之不偏估計式？

2. (5分) 在統計學的測驗中，假定測驗成績係呈變異數為29的常態分配。茲隨機抽出30位考生的成績，問此30位考生成績之變異數大於42.56的機率為何？
($\chi^2_{0.05}(29)=42.56$)

3. (20 分) *U.S. News* and *World Report* ranks colleges and universities annually. You randomly sample 100 of the national universities and liberal arts colleges from the year 2000 issue. The average cost, which includes tuition, fees, and room and board, is $24,000 with a standard deviation of $7,000.

(a) (6 分) Based on this sample, construct a 95% confidence interval of the average cost of attending a university/college in the United States.

(b) (10 分) Cost varies by quite a bit. One of the reasons may be that some universities/colleges have a better reputation than others. U.S. News and World Reports tries to measure this factor by asking university presidents and chief academic officers about the reputation of institutions. The ranking is from 1 ("marginal") to 5 ("distinguished"). You decide to split the sample according to whether the academic institution has a reputation of greater than 3.5 or not. For comparison, in 2000, Caltech had a reputation ranking of 4.7, Smith College had 4.5, and Auburn University had 3.1. This gives you the statistics shown in the accompanying table.

Reputation Category	Average Cost $\bar{Y}$ (hundred US$)	Standard Deviation of Cost (s_Y) (hundred US$)	N
Ranking > 3.5	293	36	36
Ranking ≤ 3.5	212	64	64

Test the hypothesis that the average cost for all universities/colleges is the same independent of the reputation. One-sided or two-sided hypothesis test would be appropriate to use for the alternative hypothesis?

(c) (4 分) What other factors should you consider before making a decision based on the data in (b)?

4. (25 分) 已知三組樣本其樣本的大小，平均值及變異數如下，且假設此資料適合做變異數分析

$n_1 = 3$ $\overline{X_1} = 13$ $\hat{S}_1^2 = 25$

$n_2 = 5$ $\overline{X_2} = 14$ $\hat{S}_2^2 = 16$

$n_3 = 7$ $\overline{X_3} = 15$ $\hat{S}_3^2 = 9$

(a) (5 分) 請問利用變異數分析時，資料需符合哪些假設？

(b) (5 分) 請寫出檢定的虛無假設及對立假設。

(c) (5 分) 假設顯著水準為 5%，請畫圖說明檢定的棄卻域與接受域。

(d) (10 分) 請計算並將完成以下的變異數分析(ANOVA)表。

變異來源	平方和(ss)	自由度(d.f)	均方(ms)	F 值
組間誤差	8.93			
隨機誤差				
總合				

5. (25 分) 假設一線性迴歸模型 $Y_i = \alpha + \beta X_i + \varepsilon_i$，以下抽取 5 個某國的代表性樣本，代表去年該國每人所得 X 與消費量 Y 的關係:

代表樣本	每人所得 X (千元)	每人消費量 Y (千元)
1	12	8
2	16	12
3	8	4
4	12	6
5	10	5

以上數據計算得，$\sum X = 58$，$\sum X^2 = 708$，$\sum Y = 35$，$\sum Y^2 = 285$，$\sum XY = 442$

(a) (5分) 請問為方便以OLS估計，我們需對 ε_i 做那些假設？

(b) (5分) 請證明迴歸直線通過 $(\bar{X}, \bar{Y})$。

(c) (10分) 試求迴歸直線 $\hat{Y} = \hat{\alpha} + \hat{\beta} X$。

(d) (5分) 畫圖並說明該國每人所得與消費量的關係，並說明估計係數的經濟意義。