



國立臺灣海洋大學一〇〇學年度研究所碩士班暨碩士在職專班入學考試試題

考試科目：工程數學

系所名稱：光電科學研究所碩士班不分組

1. 答案以橫式由左至右書寫。2. 請依題號順序作答。

1. (14%) 設 $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & x \end{bmatrix}$ 與 $B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & -1 \end{bmatrix}$ 相似, 求(1)x 與 y; (2)求一個滿足 $P^{-1}AP = B$ 的可

逆矩陣 P

2. (10%) 利用逆矩陣解下列線性方程組:

$$(1) \begin{cases} 1x + 2y + 3z = 1 \\ 2x + 2y + 5z = 2 \\ 3x + 5y + z = 3 \end{cases}; (2) \begin{cases} x - y - z = 2 \\ 2x - y - 3z = 1 \\ 3x + 2y - 5z = 0 \end{cases}$$

3. (10%) 已知三維向量空間 R^3 中兩個向量 $X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $Y = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ 正交, 求一個非零向量 Z, 使 X, Y, Z 兩兩正交

Z 兩兩正交

4. (10%) Evaluate $\int_0^{\infty} \frac{dx}{1+x^4}$ by contour integral.
5. (15%) Determine all values of the following roots: (a) $\sqrt[3]{1}$; (b) $\sqrt{1+i}$; (c) $(z-1)^3 + z^3 = 0$.
6. (8%) Given that $f(z) = z^2 - iz = u(x, y) + iv(x, y)$ in the complex plane. (a) Find u and v . (b) Determine whether the Cauchy-Riemann equation hold or not.
7. (10%) Solve the general solution as well as the singular solution of the following equation:

$$\frac{dy}{dx} = 8x^3 y^2.$$

8. (10%) Solve the general solution of the following equation $\frac{dy}{dx} = \frac{1}{x-y} + 1$.
9. (8%) In a one dimensional system, a particle with mass m was moving in the x -axis. The initial position and velocity of the particle were x_i and v_i , while the final position and velocity were x_f and v_f . By knowing $v \equiv \frac{dx}{dt}$ and $a \equiv \frac{dv}{dt}$, prove by integrating both sides on

$$F dx = m a dx, \text{ for showing that } F(x_f - x_i) = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2.$$

10. (5%) Let $\vec{v}(x, y, z)$ be a differentiable vector function, where x , y , and z are Cartesian coordinates in space. Given $\vec{v}(x, y, z) = 3xz\hat{i} + 2xy\hat{j} - yz^2\hat{k}$, please calculate the divergence of $\vec{v}(x, y, z)$, i.e. $\nabla \cdot \vec{v}$.