考試科目： 基礎計算機科學(含資料結構、演算法)

系所名稱： 資訊工程學系碩士班不分組

1.答案以橫式由左至右書寫。2.請依題號順序作答。

1. Explain and compare each pair of terms: (16% = 4*4%)
   (a) Tree vs. heap
   (b) Complete graph vs. bipartite graph
   (c) Recurrence relations vs. Fibonacci sequence
   (d) Hamiltonian cycle vs. Euler cycle

2. What is the worst-case time of *quicksort*? In what situation will it happen? What technique can be applied to avoid always selecting bad partition elements? Show that the expected run time of *quicksort* is $\Theta(n \lg n)$ with the above technique. What is a stable sort? Give a simple example to show that *quicksort* is not a stable sort. Show how the *quicksort* algorithm sorts the following array (in nondecreasing order) (17% = 1+1+2+5+2+2+4%)

   37, 61, 45, 27, 23, 15, 87, 19, 72, 28, 57, 6, 19

3. Let $f(n)$ and $g(n)$ be two asymptotically nonnegative functions. The asymptotic notations O, Ω, and θ are defined as follows

$O(g(n)) = \{f(n)$: there exist positive constants c and n_0 such that $0 \le f(n) \le cg(n)$ for all $n \ge n_0\}$.

$\Omega(g(n)) = \{f(n)$: there exist positive constants c and n_0 such that $0 \le cg(n) \le f(n)$ for all $n \ge n_0\}$.

$\theta(g(n)) = \{f(n)$: there exist positive constants c_1, c_2 and n_0 such that $0 \le c_1 g(n) \le f(n) \le c_2 g(n)$ for all $n \ge n_0\}$.

a. Show that $n^2 - 2n + 1 = O(n^2)$. (2%)
b. Show that $n^2 - 2n + 1 = \Omega(n^2)$. (2%)
c. Show that $n^2 - 2n + 1 = \theta(n^2)$. (2%)
d. Show that $f(n) = \theta(g(n))$ if and only if $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$. (8%)
e. What's wrong with the following argument? (8%)

$$\sum_{1 \le k \le n} k \cdot n = \sum_{1 \le k \le n} O(n) = n \cdot O(n) = O(n^2)$$

4. Given a sequence $X = (x_1, x_2,\ldots, x_n)$ the *longest increasing subsequence problem* is to find an increasing subsequence of X with the longest length.

   a. Find a longest increasing subsequence of (1, 8, 4, 12, 2, 10, 6). (4%)
   b. Describe an algorithm that solves the longest increasing subsequence problem in $O(n^2)$ time. (Hint: use dynamic programming) (8%)

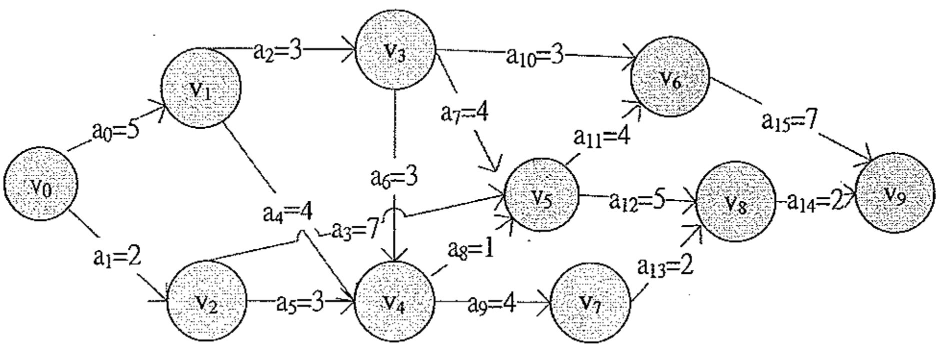
5. Let *BFS(i)* and *DFS(i)* denote the outcomes of visiting all nodes in a graph G starting from node *i* by breadth-first-search and depth-first-search respectively. Given a directed graph $G = (V, E)$ and $V(G) = \{A, B, C, D, E\}$. Please answer the following questions：(a) If BFS(A) = DFS(A), draw one possible configuration of the graph G. (b) If *BFS(A) =DFS(A), BFS(C) =DFS(C)*, and *G* is connected but *G* is not cyclic, draw one possible configuration of the graph *G*. (6%)

6. Convert the following infix formula into postfix form (according to the priority defined in ANSI C): (6%)

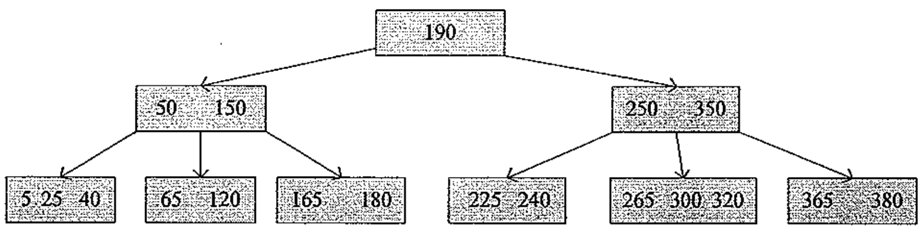
   (a) (A+B)*D +E /(F+A*D) + C (b) !(A && ! ((B < C) || (C > D))) || (C > E)

7. If a forest has *n* vertices and *m* trees, prove that it has *n-m* edges. (3%)

8. For the AOE network of the following figure, what are the early and late times of each activity ? In addition, what activities are critical ? (8%)



9. Given a B-tree as the following figure. Please redraw this B-tree if the node '250' is deleted. (5%)



10. Ordering by asymptotic growth rates (5%)

(a) $n^{\log\log n}$ (b) $n!$ (c) $(1.001)^{(\log n)^3}$ (d) $1000000\log\log n$ (e) $0.001n^{0.0001}$ (f) $4^{\log n}$ (g) $\left(\frac{4}{3}\right)^n$