國立臺灣海洋大學一００學年度研究所碩士班暨碩士在職專班入學考試試題

考試科目： 計算機數學(含線性代數、離散數學)

系所名稱： 資訊工程學系碩士班不分組

1.答案以橫式由左至右書寫。2.請依題號順序作答。

I. Let I_n be the $n \times n$ identity matrix.

(i) Find a 1×2 matrix X and a 2×1 matrix Y such that $XY = I_1 = [1]$. (5 分)

(ii) Is it possible to find matrices X and Y that satisfy (i) and also satisfy

$$YX = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$? why or why not? (5 分)

II. Suppose that $\vec{b}_1, \vec{b}_2, \cdots \vec{b}_k$ is a linearly independent set of vectors in R^m and suppose A is an $m \times n$ matrix. If $\vec{x}_1, \vec{x}_2, \cdots, \vec{x}_k$ are each solutions to the respective equation

$$A\vec{x}_1 = \vec{b}_1, \cdots, A\vec{x}_k = \vec{b}_k$$

show that $\vec{x}_1, \vec{x}_2, \cdots, \vec{x}_k$ are linearly independent vectors in R^n. (10 分)

III. Consider vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ in R^4; we are told that $\vec{v}_i \cdot \vec{v}_j$ is the entry a_{ij} of the matrix A given below

$$A = \begin{bmatrix} 3 & 5 & 11 \\ 5 & 9 & 20 \\ 11 & 20 & 49 \end{bmatrix}$$

(i) Find $\|\vec{v}_1 + \vec{v}_2\|$. (5 分)

(ii) Find the orthogonal projection of $\vec{v}_1$ onto the space spanned by $\vec{v}_2$ and $\vec{v}_3$. Express your answer as a linear combination of $\vec{v}_2$ and $\vec{v}_3$. (5 分)

IV. Consider a dynamical system

$$\vec{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

whose transformation from time t to t + 1 is given by the following equations

$$x_1(t+1) = 0.1x_1(t) + 0.2x_2(t) + 1$$
$$x_2(t+1) = 0.4x_1(t) + 0.3x_2(t) + 2$$

(i) Find a 2×2 matrix A and a vector $\vec{b}$ in R^2 such that

$$\vec{x}(t+1) = A\vec{x}(t) + \vec{b}.$$ (5 分)

(ii) Let $\vec{y}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ 1 \end{bmatrix}$. Find a 3×3 matrix B such that

$$\vec{y}(t+1) = B\vec{y}(t).$$ (5 分)

(iii) Find a 3×3 diagonal matrix Λ and an invertible 3×3 matrix S such that $B = S\Lambda S^{-1}$. (5 分)

(iv) Find $\lim_{t\to\infty} \vec{x}(t)$ if $\vec{x}(0) = \begin{bmatrix} 50 \\ 100 \end{bmatrix}$. (5 分)

V. Use combinatorial method to prove. (20 分)

(a) $\sum_{k=1}^{n} k\binom{n}{k} = n * 2^{n-1}$

(b) $\sum_{k=1}^{n} k\binom{n}{k}^2 = n\binom{2n-1}{n-1}$

VI. How many students must be in a class to guarantee that at least 7 were born on the same day of the week? (10 分)

VII. Use generating function to solve $a_k = 5 * a_{k-1}$, k = 1,2,3,……

With $a_0 = 7$.(20 分)