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A fuzzy AHP approach based on the concept of possibility extent

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Abstract The main purpose of this paper is to develop a fuzzy AHP method for tackling the uncertainty and imprecision existing in multi-criteria decision process. The proposed method uses fuzzy pair-wise comparison judgments in place of exact numerical values of the comparison ratios. The geometric mean technique is used to integrate all decision-makers' opinions and construct the fuzzy positive reciprocal matrices. The algebraic operations of triangular fuzzy numbers are utilized to calculate the fuzzy suitability indices of all alternatives. The extent analysis method is used to compute the degree of possibility of priority among fuzzy suitability indices. Besides, two principles are presented to solve the multi-criteria decision problem in a fuzzy decision environment. Principle I provides a partial preorder, and Principle II gives a total preorder on the set of the possible alternatives. Finally, a numerical example of selecting the company with optimal performance in performing customer relationship management is used to demonstrate the decision process of proposed method.

Keywords Fuzzy set theory · Triangular fuzzy number · Linguistic values · Fuzzy AHP · Extent analysis method

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1 Introduction

In general, the selection/evaluation of alternatives is a multi-criteria decision making (MCDM) problem. In the past, many MCDM methods for selecting/evaluating alternatives have been developed (e.g. Guitouni and Martel 1998; Ensslin et al. 2000; Fenton and Neil 2001; Borges and Villavicencio 2004; Hodgkin et al. 2005; Neves et al. 2009). The analytic hierarchy process (AHP) has been widely used to address the multi-criterion decision making problems in real situations (Saaty 1980). In conventional AHP, the pair-wise comparison is established using a nine-point scale which converts the human preferences between available alternatives as equally, moderately, strongly, very strongly or extremely preferred. Despite its popularity and simplicity in concept, it has been generally criticized that it is not sufficient to take into account the uncertainty and ambiguity associated with the mapping of one's perception to a discrete scale (Kwong and Bai 2003). Due to the real situations in which information is incomplete or imprecise, and views are subjective or expressing with linguistic characteristics creating a fuzzy decision-making environment. The linguistic assessment of human's feelings and judgements are vague and it is difficult to represent as crisp numbers. It feels more confident to use interval judgements or fuzzy evaluations than fixed value (Chan and Kumar 2007). Hence, fuzzy numbers or linguistic values characterized by fuzzy numbers are used to convey the assessments of human's feelings and judgements. The synthetic extent analysis method of fuzziness-based analytic hierarchy process (AHP) approach so-called as fuzzy extended AHP (FEAHP), proposed by Chang in (1996), is designed to minimize such adverse conditions and strengthen the choice process and decide the final priority results. The FEAHP uses the triangular fuzzy numbers as a pair-wise comparison scale to derive the weights of criteria and the appropriateness values of all alternatives versus to each criterion. The final scores of all alternatives are calculated and based on that resulted the optimal alternative. This approach can adequately handle the inherent uncertainty and imprecision of the human decision making process and provides the flexibility and robustness needed for the decision maker to understand the decision problem. These merits of the approach would facilitate its use in real-life situations for making effective decisions (Chan and Kumar 2007).

The fuzzy AHP method has been applied to many different fields. Weck et al. (1997) used the extended fuzzy AHP method to evaluate alternative production cycles. Sohn et al. (2001) proposed a fuzzy AHP approach to analyze six options for spent fuel management in Korea. Kuo et al. (2002) developed a decision support system using the fuzzy AHP for selecting convenience store location. Bozdağ et al. (2003) used fuzzy group decision making for selection among computer integrated manufacturing systems. Murtaza (2003) presented a fuzzy version of AHP to country risk assessment problem. Kahraman et al. (2004) developed an analytical tool using fuzzy AHP to select the best catering service firm. Ayağ (2005) presented a fuzzy AHP-based simulation approach to evaluate conceptual design alternatives in a new product development environment. Ayağ and Özdemir (2006) presented a fuzzy AHP approach to evaluating machine tool alternative. Chan and Kumar (2007) used fuzzy extended AHP-based approach to consider global supplier development risk factors. Srdjevic and Medeiros (2008) presented a fuzzy AHP methodology for the assessment of water management plans in part of the Paraguacu River Basin in Brazil. Chen and Chen (2008) improved the advantages of traditional manufacturing organizations in Taiwan based on vision using fuzzy AHP. Perçin (2008) used fuzzy AHP for evaluating the benefits of information-sharing decisions in a supply chain. Li and Chen (2009) evaluated the architectural design services by using fuzzy AHP. Arslan (2009) presented a hybrid model of fuzzy and AHP for handling public assessments on transportation projects. Kannan (2009) proposed a fuzzy approach for the selection of third party reverse logistics provider.

Lee et al. (2009) presented a quantified SWOT analytical method that integrated the method of fuzzy AHP method, to provide more detailed and quantified data for SWOT environmental analysis to assess the competitive relation for locations develop different types' global logistics hub in Pacific–Asia region. Tsai et al. (2010) utilized fuzzy hierarchy sensitive with Delphi method to evaluate hospital organization performance. Kahraman and Kaya (2010) proposed a fuzzy multi-criteria methodology for selection among energy alternatives. Aydin and Arslan (2010) utilized Fuzzy AHP to find the optimal hospital location, etc.

Although the fuzzy AHP methods stated above are applicable to widely MCDM problems, however, some improvements would be valuable. For example, using geometric mean technique to construct the fuzzy positive reciprocal matrices (Aczel and Saaty 1983; Uppuluri 1978, 1983; Buckley 1985) and to characterize the multiple decision-makers' consensus opinions (Saaty 1980), considering the indifference among alternatives when all decision criteria are integrated consideration (Brans and Vincke 1985). In this paper, according to the above mentioned concepts from the literatures, a new fuzzy AHP method based on the concept of possibility extent is proposed.

The remainder of this paper is organized as follows. The following section presents the research methodology. The new fuzzy AHP method is proposed in Sect. 3. In Sect. 4, a numerical example applying the proposed fuzzy AHP method is presented. Finally, conclusion is made in the last section.

2 Research methodology

In this section, some of the concepts used in this paper are briefly introduced.

2.1 Fuzzy set theory

In general, the thinking process, logic inference, and cognition to the surrounding environment are often vague and uncertain. Therefore the traditional analytic methods which offered crisp value results often fail to adequate for real life scenarios which are often ambiguous and uncertain. Fuzzy set theory (Zadeh 1965), the fuzzy numbers and its operation algorithm (Dubois and Prade 1978) can be efficiently utilized to characterize ambiguous and vague data expression and transmission.

2.2 Triangular fuzzy numbers

In a universe of discourse X, a fuzzy subset A of X is defined by a membership function $f_A(x)$, which maps each element x in X to a real number in the interval [0, 1]. The function value $f_A(x)$ represents the grade of membership of x in A.

A fuzzy number A (Dubois and Prade 1978) in real line $\Re$ is a triangular fuzzy number if its membership function $f_A : \Re \to [0, 1]$ is

$$f_A(x) = \begin{cases} (x-c)/(a-c), & c \le x \le a \\ (x-b)/(a-b), & a \le x \le b \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

with $-\infty < c \le a \le b < \infty$. The triangular fuzzy number can be denoted by (c, a, b).

The parameter a gives the maximal grade of $f_A(x)$, i.e., $f_A(a) = 1$; it is the most probable value of the evaluation data. In addition, 'c' and 'b' are the lower and upper bounds of the available area for the evaluation data. They are used to reflect the fuzziness of the evaluation data. The narrower the interval $[c, b]$, the lower the fuzziness of the evaluation data. Here, the triangular fuzzy numbers are used to denote the approximate reasoning of linguistic values. They are used to convey the subjective evaluation of the decision-makers. The reason for using a triangular fuzzy number is that decision-makers find it intuitively easy to use. For example, the linguistic value "weakly more important" can be represented by (1, 3/2, 2). In addition, the non-fuzzy number, an exact number, 'a' can be represented by (a, a, a). For example, 'a value of 600' can be represented by (600, 600, 600). The linguistic value "Just equal" can be represented by (1, 1, 1).

2.3 The algebraic operations of fuzzy numbers

Let $A_1 = (c_1, a_1, b_1)$ and $A_2 = (c_2, a_2, b_2)$ be fuzzy numbers. According to the extension principle (Zadeh 1965) the algebraic operations of any two fuzzy numbers A_1 and A_2 can be expressed as

(1) Fuzzy addition $\oplus$:

$$A_1 \oplus A_2 = (c_1 + c_2, a_1 + a_2, b_1 + b_2);$$

(2) Fuzzy multiplication $\otimes$:

$$k \otimes A_2 = (kc_2, ka_2, kb_2), \quad k \in \Re, \quad k \geq 0;$$
$$A_1 \otimes A_2 \cong (c_1 c_2, a_1 a_2, b_1 b_2), \quad c_1 \geq 0, \quad c_2 \geq 0.$$

(3) Fuzzy division $\varnothing$:

$$(A_1)^{-1} = (c_1, a_1, b_1)^{-1} \cong (1/b_1, 1/a_1, 1/c_1), \quad c_1 > 0;$$
$$A_1 \varnothing A_2 \cong (c_1/b_2, a_1/a_2, b_1/c_2), \quad c_1 \geq 0, \quad c_2 > 0.$$

(4) Fuzzy root:

$$A_1^{1/n} \cong \left(c_1^{1/n}, a_1^{1/n}, b_1^{1/n}\right), c_1 \geq 0, n > 1, n \in N.$$

2.4 Linguistic values

A linguistic variable (Zadeh 1975) is a variable whose values are expressed in words of natural language. For example, "weight" is a linguistic variable, with values of Very Low, Low, Medium, High, Very high, etc. Linguistic value can be represented by the approximate reasoning of fuzzy set theory. In this paper, linguistic values characterized by triangular fuzzy numbers are utilized to describe the relative importance between criteria and the relative suitability between alternatives versus various evaluation criteria.

3 Proposed fuzzy AHP

The fuzzy AHP model based on the concept of possibility extent presented herein can be summarized as follows:

Step 1: Form a committee of decision-makers and, then select the evaluation criteria and identify the prospective alternatives.

Step 2: Choose preference ratings for describing the relative importance between criteria and the relative suitability between alternatives versus various criteria.

Table 1 The fuzzy linguistic scale

Linguistic values	Definition	Triangular fuzzy numbers
JE	Just equal	(1, 1, 1)
EI	Equally important / appropriateness	(1/2, 1, 3/2)
WMI	Weakly more important / appropriateness	(1, 3/2, 2)
SMI	Strongly more important / appropriateness	(3/2, 2, 5/2)
VSMI	Very strongly more important / appropriateness	(2, 5/2, 3)
AMI	Absolutely more important / appropriateness	(5/2, 3, 7/2)

Step 3: Build fuzzy pair-wise comparison matrices of all criteria and all alternatives versus various criteria, respectively.
Step 4: Calculate the values of fuzzy synthetic extent of all criteria, and alternatives versus various criteria, respectively.
Step 5: Aggregate the weightings of criteria and fuzzy ratings of alternatives versus all criteria and obtain the fuzzy suitability indices.
Step 6: Calculate the degree of possibility of priority among fuzzy suitability indices.
Step 7: Calculate the domination flows and weakness flows of all alternatives.
Step 8: Choose an optimal alternative or a set of best alternatives.

3.1 Choose the preference rating system

Two preference ratings may be used. They are fuzzy numbers and linguistic values. Based on the practical needs, the decision-makers may apply one or both of them.

The importance weight of each criterion can be obtained by either directly assigning weight or indirectly using pair-wise comparisons. In here, it is suggested that the decision-makers employ the relative importance set (Kahraman et al. 2006) W = {Just equal (JE), Equally important (EI), Weakly more important (WMI), Strongly more important (SMI), Very strongly more important (VSMI), Absolutely more important (AMI)} that are expressed in triangular fuzzy numbers as shown in Table 1, to evaluate the relative importance between criteria. Similarly, the relative appropriateness set S is defined as set W only the linguistic value "important" is replaced by "appropriateness". The linguistic values in set S are used by decision-makers to evaluate the relative appropriateness between alternatives versus some one criterion.

3.2 Build fuzzy pair-wise comparison matrices of all criteria and all alternatives versus various criteria

Collecting pair-wise comparison matrices of each layer to represent the relative importance of criteria or relative appropriateness of alternatives versus various criteria is an important step in fuzzy AHP method. Consequently, the experts evaluate the relative importance or relative appropriateness by utilized the linguistic values in relative important set W or in advantage strength set S and are transformed into triangular fuzzy numbers, as in Table 1. Owing to the geometric mean is more effective in representing the multiple decision-makers' consensus opinions (Saaty 1980). This paper would like to use geometric mean approach instead of arithmetic mean approach.

Let $M_{ij}^{h} = \left(c_{ij}^{h}, a_{ij}^{h}, b_{ij}^{h}\right), h = 1, 2, \ldots, n, \quad \forall i, j = 1, 2, \ldots, k$, be the relative importance or relative suitability given to criterion/alternative i to criterion/alternative j by expert h on some one criteria layer or alternative layer in the hierarchy. Then, the pair-wise comparison matrix is defined as $[M_{ij}]_{k \times k}$. After integrating the opinions of all n experts, the fuzzy pair-wise comparison matrix (given to criterion/alternative i to criterion/alternative j) for the criteria/alternative layer, can be denoted by

$$M = \left[M_{ij}\right]_{k \times k} = \begin{bmatrix} \tilde{1} & M_{12} & \cdots & M_{1k} \\ 1/M_{12} & \tilde{1} & \cdots & M_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ 1/M_{1k} & 1/M_{2k} & \cdots & \tilde{1} \end{bmatrix}$$

where

$$M_{ij} \cong \left(c_{ij}, a_{ij}, b_{ij}\right) = \left(\left(\prod_{h=1}^{n} c_{ij}^{h}\right)^{1/n}, \left(\prod_{h=1}^{n} a_{ij}^{h}\right)^{1/n}, \left(\prod_{h=1}^{n} b_{ij}^{h}\right)^{1/n}\right), \quad \forall i \neq j, \tag{2}$$

$$\tilde{1} = (1, 1, 1),$$

$$M_{ij} \otimes M_{ji} \cong (1, 1, 1).$$

3.3 Calculate the values of fuzzy synthetic extent of all criteria and alternatives versus various criteria

By replacing the fuzzy arithmetic mean by fuzzy geometric mean in Chang (1996) extent analysis method on fuzzy AHP, the value of fuzzy synthetic extent with respect to the ith criterion/alternative is defined as

$$W_i = G_i \otimes \left(\sum_{i=1}^{k} G_i\right)^{-1}, \quad i = 1, 2, \ldots, k \tag{3}$$

Where

$$G_i = \left(\prod_{j=1}^{k} M_{ij}\right)^{1/k} \cong (c_i, a_i, b_i) = \left(\left(\prod_{j=1}^{k} c_{ij}\right)^{1/k}, \left(\prod_{j=1}^{k} a_{ij}\right)^{1/k}, \left(\prod_{j=1}^{k} b_{ij}\right)^{1/k}\right)$$

$$\left(\sum_{i=1}^{k} G_i\right)^{-1} \cong \left(1/\sum_{i=1}^{k} b_i, 1/\sum_{i=1}^{k} a_i, 1/\sum_{i=1}^{k} c_i\right)$$

3.4 Aggregate the weightings of criteria and fuzzy ratings of alternatives versus all criteria and obtain the fuzzy suitability indices

Let $W_t = (c_t, a_t, b_t), t = 1, 2, \ldots, k$, be the value of fuzzy synthetic extent with respect to criterion C_t, and $S_{it} = (q_{it}, o_{it}, p_{it}), i = 1, 2, \ldots, m; t = 1, 2, \ldots, k$, be the value of fuzzy synthetic extent of alternative A_i versus criterion C_t. Further, S_{it} and W_t are aggregated by averaging the corresponding alternatives versus all criteria. The fuzzy suitability index of the ith alternative can be obtained by standard arithmetic method.

$$F_i = [(S_{i1} \otimes W_1) \oplus (S_{i2} \otimes W_2) \oplus \cdots \oplus (S_{ik} \otimes W_k)] \tag{4}$$

By the extension principle, F_i can be expressed by

$$F_i \cong (f_i, d_i, e_i)$$

Where

$$f_i = \sum_{t=1}^{k} q_{it} c_t, \quad d_i = \sum_{t=1}^{k} o_{it} a_t, \quad e_i = \sum_{t=1}^{k} p_{it} b_t.$$

3.5 Calculate the degree of possibility of priority among fuzzy suitability indices

Calculating the degree of possibility of priority between two fuzzy numbers is another important step in Chang's method (1996). This step can obtain to compare which is the greatest fuzzy value among several fuzzy numbers.

Let $F_i = (f_i, d_i, e_i)$ and $F_j = (f_j, d_j, e_j)$ be fuzzy numbers, then the degree of possibility of $F_j \geq F_i$ is defined as $V\left(F_j \geq F_i\right) = \sup_{y \geq x}[\min(f_{F_i}(x), f_{F_j}(y))]$. The degree of possibility of $F_j \geq F_i$ can be also expressed as

$$V(F_j \geq F_i) = \begin{cases} 1, & \text{if } d_j \geq d_i \\ 0, & \text{if } f_i \geq e_j \\ \frac{f_i - e_j}{(d_j - e_j) - (d_i - f_i)}, & \text{otherwise} \end{cases} \tag{5}$$

In order to compare F_i and F_j, we need both the values of $V(F_i \geq F_j)$ and $V(F_j \geq F_i)$.

By utilizing Eq. 5, the degree of possibility of priority among fuzzy suitability indices can be easily calculated.

3.6 Choose the most suitable alternative

Integrating the degree of possibility of priority among fuzzy suitability indices into the concept used in PROMETHEE Methods (Brans and Vincke 1985), two methods can be utilized to obtain a most suitable alternative or a set of best alternatives in a fuzzy decision-making environment.

(1) Principle I: *Ranking the alternative by a partial preorder*
Define the domination flow of a alternative number F_i $(i = 1, 2, \ldots, m)$, denoted by $D(F_i)$, as follow:

$$D(F_i) = \sum_{j=1, j \neq i}^{m} V\left(F_i \geq F_j\right). \tag{6}$$

The weakness flow of a alternative number F_i $(i = 1, 2, \ldots, m)$, denoted by $E(F_i)$, can be defined by

$$E(F_i) = \sum_{j=1, j \neq i}^{m} V\left(F_j \geq F_i\right) \tag{7}$$

The larger $D(F_i)$, the more F_i dominates the other alternatives. The smaller $E(F_i)$, the less F_i is dominated.

Define the two total preorders (P^+, I^+) and (P^-, I^-) such that

$$\begin{aligned} F_i P^+ F_j &\quad \text{iff} \quad D(F_i) > D\left(F_j\right), \\ F_i P^- F_j &\quad \text{iff} \quad E(F_i) < E\left(F_j\right); \\ F_i I^+ F_j &\quad \text{iff} \quad D(F_i) = D\left(F_j\right), \\ F_i I^- F_j &\quad \text{iff} \quad E(F_i) = E\left(F_j\right) \end{aligned}$$

We obtain the following partial preorder by considering their intersection:

$$F_i \text{ dominates } F_j \left(\text{denoted by } F_i P^{(1)} F_j\right) \text{ if } \begin{cases} F_i P^+ F_j \text{ and } F_i P^- F_j \\ \text{or} \\ F_i P^+ F_j \text{ and } F_i I^- F_j \\ \text{or} \\ F_i I^+ F_j \text{ and } F_i P^- F_j \end{cases}$$

$$F_i \text{ is indifferent to } F_j \left(\text{denoted by } F_i I^{(1)} F_j\right) \text{ if } F_i I^+ F_j \text{ and } F_i I^- F_j,$$

$$F_i \text{ and } F_j \text{ are incomparable } \left(\text{denoted by } F_i IR F_j\right) \text{ otherwise.}$$

(2) Principle II: *Ranking the alternatives by a total preorder*
Assume that a total preorder has been considered by the decision-maker. The Principle II can be used.
Define the net flow

$$N(F_i) = D(F_i) - E(F_i). \tag{8}$$

The net flow can be utilized to rank the alternatives:

$$F_i \text{ dominates } F_j \left(\text{denoted by } F_i\ P^{(2)} F_j\right) \text{ iff } N(F_i) > N\left(F_j\right),$$

$$F_i \text{ is indifferent to } F_j \left(\text{denoted by } F_i I^{(2)} F_j\right) \text{ iff } N(F_i) = N\left(F_j\right).$$

4 Numerical example

Without any doubt, facing competitive business activities, the customer relationship management is one of the main marketing issues nowadays and it has been in recent years, customer retention is a valuable strategy to ensure long-term profitability and success of the company (Buckinx and Poel 2005). In this section, an application of performance evaluation of performing customer relationship management is developed to demonstrate the decision process of proposed fuzzy AHP. The process of the algorithm is empirically implemented, step by step, as follows.

Step 1 Suppose that a company needs to choose a benchmarking in performing customer relationship management. After an initial screening, three companies A_1, A_2, and A_3 are left for further evaluation. A committee of three decision-makers, i.e. D_1, D_2 and D_3, has been formed to determine the most suitable company. Referring to Körner and Zimmermann (2000) and considering the ease of computation, three selection criteria are used.

(1) value-added (C_1);
(2) customer interaction (C_2);
(3) customer profiling (C_3)

Table 2 The fuzzy pairwise comparison of performance criteria

	C$_1$	**C**$_2$	**C**$_3$
C$_1$	(1, 1, 1)	$\left(\frac{3}{2}, 2, \frac{5}{2}\right)$	$\left(\frac{3}{2}, 2, \frac{5}{2}\right)$ $\left(\frac{5}{2}, 3, \frac{7}{2}\right)$ $\left(\frac{1}{2}, 1, \frac{3}{2}\right)$
C$_2$	$\left(\frac{2}{5}, \frac{1}{2}, \frac{2}{3}\right)$	(1,1,1)	$\left(\frac{1}{2}, 1, \frac{3}{2}\right)$
C$_3$	$\left(\frac{2}{5}, \frac{1}{2}, \frac{2}{3}\right)$ $\left(\frac{2}{7}, \frac{1}{3}, \frac{2}{5}\right)$ $\left(\frac{2}{3}, 1, 2\right)$	$\left(\frac{2}{3}, 1, 2\right)$	(1, 1, 1)

Table 3 The fuzzy pairwise comparison of all alternatives versus criteria C_1

C$_1$	**A**$_1$	**A**$_2$	**A**$_3$
A$_1$	(1,1, 1)	$\left(2, \frac{5}{2}, 3\right)$ $\left(\frac{1}{3}, \frac{2}{5}, \frac{1}{2}\right)$ $\left(\frac{2}{3}, 1, 2\right)$	$\left(1, \frac{3}{2}, 2\right)$ $\left(\frac{2}{3}, 1, 2\right)$ $\left(\frac{2}{3}, 1, 2\right)$
A$_2$	$\left(\frac{1}{3}, \frac{2}{5}, \frac{1}{2}\right)$ $\left(2, \frac{5}{2}, 3\right)$ $\left(\frac{1}{2}, 1, \frac{3}{2}\right)$	(1,1,1)	$\left(1, \frac{3}{2}, 2\right)$
A$_3$	$\left(\frac{1}{2}, \frac{2}{3}, 1\right)$ $\left(\frac{1}{2}, 1, \frac{3}{2}\right)$ $\left(\frac{1}{2}, 1, \frac{3}{2}\right)$	$\left(\frac{1}{2}, \frac{2}{3}, 1\right)$	(1,1,1)

Table 4 The fuzzy pairwise comparison of all alternatives versus criteria C_2

C$_2$	**A**$_1$	**A**$_2$	**A**$_3$
A$_1$	(1,1,1)	$\left(\frac{3}{2}, 2, \frac{5}{2}\right)$	$\left(\frac{2}{5}, \frac{1}{2}, \frac{2}{3}\right)$ $\left(\frac{2}{3}, 1, 2\right)$ $\left(\frac{1}{2}, 1, \frac{3}{2}\right)$
A$_2$	$\left(\frac{2}{5}, \frac{1}{2}, \frac{2}{3}\right)$	(1,1,1)	$\left(\frac{2}{7}, \frac{1}{3}, \frac{2}{5}\right)$
A$_3$	$\left(\frac{3}{2}, 2, \frac{5}{2}\right)$ $\left(\frac{1}{2}, 1, \frac{3}{2}\right)$ $\left(\frac{2}{3}, 1, 2\right)$	$\left(\frac{5}{2}, 3, \frac{7}{2}\right)$	(1,1,1)

Step 2 The decision-makers employ a linguistic values set W (or S), shown in Table 1, to evaluate the relative importance between criteria and the relative appropriateness between alternatives versus some one criterion.

Step 3 By using fuzzy pair-wise comparison, the fuzzy evaluation matrix, relevant to the objective, is constructed (Table 2). Similarly, the committee compares alternatives A_1, A_2 and A_3 under each of the criteria separately. The results are shown in Tables 3, 4, and 5. Then, by formula (2), we can get fuzzy pair-wise comparison matrices of all criteria and all alternatives versus various criteria. The results are shown in Tables 6, 7, 8, and 9.

Table 5 The fuzzy pairwise comparison of all alternatives versus criteria C_3

$\mathbf{C_3}$	$\mathbf{A_1}$	$\mathbf{A_2}$	$\mathbf{A_3}$
$\mathbf{A_1}$	(1,1,1)	$\left(1, \frac{3}{2}, 2\right)$	$\left(2, \frac{5}{2}, 3\right)$ $\left(\frac{3}{2}, 2, \frac{5}{2}\right)$ $\left(2, \frac{5}{2}, 3\right)$
$\mathbf{A_2}$	$\left(\frac{1}{2}, \frac{2}{3}, 1\right)$	(1,1,1)	$\left(1, \frac{3}{2}, 2\right)$
$\mathbf{A_3}$	$\left(\frac{1}{3}, \frac{2}{5}, \frac{1}{2}\right)$ $\left(\frac{2}{5}, \frac{1}{2}, \frac{2}{3}\right)$ $\left(\frac{1}{3}, \frac{2}{5}, \frac{1}{2}\right)$	$\left(\frac{1}{2}, \frac{2}{3}, 1\right)$	(1,1,1)

Table 6 The fuzzy pair-wise comparison matrices of all criteria

	C_1	C_2	C_3
C_1	(1, 1, 1)	(1.5, 2, 2.5)	(1.23, 1.82, 2.36)
C_2	(0.4, 0.5, 0.67)	(1, 1, 1)	(0.5, 1, 1.5)
C_3	(0.42, 0.55, 0.81)	(0.67, 1, 2)	(1, 1, 1)

Table 7 The fuzzy pair-wise comparison matrices of all alternatives versus criterion C_1

C_1	A_1	A_2	A_3
A_1	(1, 1, 1)	(0.76, 1, 1.44)	(0.76, 1.14, 2)
A_2	(0.69, 1, 1.31)	(1, 1, 1)	(1, 1.5, 2)
A_3	(0.5, 0.87, 1.31)	(0.5, 0.67, 1)	(1, 1, 1)

Table 8 The fuzzy pair-wise comparison matrices of all alternatives versus criterion C_2

C_2	A_1	A_2	A_3
A_1	(1, 1, 1)	(1.5, 2, 2.5)	(0.51, 0.79, 1.26)
A_2	(0.4, 0.5, 0.67)	(1, 1, 1)	(0.29, 0.33, 0.4)
A_3	(0.79, 1.26, 1.96)	(2.5, 3, 3.5)	(1, 1, 1)

Step 4 By using formula (3), the fuzzy synthetic extent of all criteria and alternatives versus various criteria can be obtained. The results are shown in Tables 10, 11, 12, and 13.

Step 5 By using formula (4), the fuzzy suitability indices of all alternatives can be obtained (see Table 14).

Step 6 By using formula (5), the degrees of possibility of priority among all fuzzy suitability indices can be obtained (see Table 15).

Step 7 Let us first assume that a partial relation would be useful to the decision-maker. The Principle I technique can be used. According Eqs. 6 and 7 we complete Table 16. It is then easy to obtain the preorders P^{+} and P^{-}, the intersection of which is:

$$F_1 P^{(1)} F_2, \quad F_1 P^{(1)} F_3, \text{ But, } F_2 I R F_3$$

So that the partial Principle I preorder shows:

$$A_1 \text{ dominates } A_2, A_1 \text{ dominates } A_3, \text{ But, } A_2 \text{ and } A_3 \text{ are incomparable.}$$

Now, suppose that the decision-maker wants a total preorder, we then can use Principle II. According Eq. 8 we can compute the net flows:

$$N(F_1) = 0.2132, N(F_2) = -0.1076, N(F_3) = -0.1056,$$

So that the partial Principle II preorder shows:

$$A_1 P^{(2)} A_3, \text{ and } A_3 P^{(2)} A_2.$$

That is, A_1 dominates A_3, and A_3 dominates A_2.

Table 9 The fuzzy pair-wise comparison matrices of all alternatives versus criterion C_3

C_3	A_1	A_2	A_3
A_1	(1, 1, 1)	(1, 1.5, 2)	(1.82, 2.32, 2.82)
A_2	(0.5, 0.67, 1)	(1, 1, 1)	(1, 1.5, 2)
A_3	(0.35, 0.43, 0.55)	(0.5, 0.67, 1)	(1, 1, 1)

Table 10 The values of fuzzy synthetic extent with respect to criterion C_t

W_t	(c_t, a_t, b_t)
W_1	(0.3083, 0.488, 0.7319)
W_2	(0.1469, 0.2519, 0.4051)
W_3	(0.1648, 0.2601, 0.476)

Table 11 The values of fuzzy synthetic extent of alternative A_i versus criterion C_1

S_{it}	(q_{i1}, o_{i1}, p_{i1})
S_{11}	(0.2143, 0.3457, 0.6057)
S_{21}	(0.2271, 0.3783, 0.5867)
S_{31}	(0.1617, 0.276, 0.4656)

Table 12 The values of fuzzy synthetic extent of alternative A_i versus criterion C_2

S_{it}	(q_{i2}, o_{i2}, p_{i2})
S_{12}	(0.2283, 0.3562, 0.5517)
S_{22}	(0.1211, 0.1681, 0.2423)
S_{32}	(0.3135, 0.4757, 0.7148)

Table 13 The values of fuzzy synthetic extent of alternative A_i versus criterion C_3

S_{it}	(q_{i3}, o_{i3}, p_{i3})
S_{13}	(0.3161, 0.4773, 0.6914)
S_{23}	(0.2056, 0.3149, 0.4892)
S_{33}	(0.1455, 0.2078, 0.3182)

Table 14 The fuzzy suitability indices of alternative A_i

F_i	(f_i, d_i, e_i)
F_1	(0.1517, 0.3826, 0.9959)
F_2	(0.1217, 0.3089, 0.7603)
F_3	(0.1199, 0.3086, 0.7818)

Table 15 The degree of possibility of priority among fuzzy suitability indices

$F_i \geq F_j$	The degree of possibility $V(F_i \geq F_j)$
$F_1 \geq F_2$	1
$F_1 \geq F_3$	1
$F_2 \geq F_1$	0.8919
$F_2 \geq F_3$	1
$F_3 \geq F_1$	0.8949
$F_3 \geq F_2$	0.9996

Table 16 The Domination, Weakness, and Net flow of a alternative number F_i

F_i	The Domination flow $D(F_i)$	The Weakness flow $E(F_i)$
F_1	2	1.7868
F_2	1.8919	1.9996
F_3	1.8944	2

5 Conclusion

This paper presents a new fuzzy AHP method, based on the concept of possibility extent, to solve the multi-criteria decision-making problem under fuzzy environment. To efficiently grip the ambiguity exist in available information and the essential fuzziness in human judgment and preference, the linguistic values characterized by triangular fuzzy numbers are utilized to evaluate the relative importance between criteria and the relative suitability between alternatives versus various criteria. To relevantly convey the representation and comprehension of all decision-makers' opinions, the geometric mean operations is utilized to aggregate the individual opinions characterized by triangular fuzzy numbers. The algebraic operations of triangular fuzzy numbers are utilized to calculate the fuzzy suitability indices of all alternatives. The extent analysis method is used to compute the degree of possibility of priority among fuzzy suitability indices. Furthermore, by integrating the degree of possibility of priority among fuzzy suitability indices into the concept, used in PROMETHEE Methods, two principles are proposed to obtain an optimal alternative or a set of best alternatives in a fuzzy decision-making environment. In summary, the proposed method considers and integrates some important viewpoints, such as using the geometric mean technique to integrate all decision-makers' opinions and construct the fuzzy positive reciprocal matrices, permitting the indifference existing among alternatives. In addition, an example regarding optimal company selection for performing customer relationship management was conducted to examine the applicability of proposed method. Although the proposed method is illustrated by company selection of optimizing customer relationship management, it can also be applied in similar selection problems, such as project selection, international distribution center selection, and many other areas of management decision or strategy selection problems.

References

Aczel, J., Saaty, T.L.: Procedures for synthesizing ratio judgments. J. Math. Psychol. **27**(1), 93–102 (1983)

Arslan, A.: A hybrid model of fuzzy and AHP for handling public assessments on transportation projects. Transportation **36**(1), 97–112 (2009)

Ayağ, Z.: A fuzzy AHP-based simulation approach to concept evaluation in a NPD environment. IIE Trans. **37**(9), 827–842 (2005)
Ayağ, Z., Özdemir, R.G.: A fuzzy AHP approach to evaluating machine tool alternatives. J. Intell. Manuf. **17**(2), 179–190 (2006)
Aydin, O., Arslan, G.: Optimal hospital location with fuzzy AHP. Bus. Rev. Cambr. **15**(1), 262–268 (2010)
Borges, P.C., Villavicencio, A.: Avoiding academic and decorative planning in GHG emissions abatement studies with MCDA: the Peruvian case. Eur. J. Oper. Res. **152**(3), 641–654 (2004)
Bozdağ, C.E., Kahraman, C., Ruan, D.: Fuzzy group decision making for selection among computer integrated manufacturing systems. Comput. Ind. **51**, 13–29 (2003)
Brans, J.P., Vincke, P.H.: A preference ranking organization method: (The PROMETHEE method for multiple criteria decision-making). Manag. Sci. **31**(6), 647–656 (1985)
Buckinx, W., Poel, D.V.: Customer base analysis: partial defection of behaviourally loyal clients in a non-contractual FMCG retail setting. Eur. J. Oper. Res. **164**(1), 252–268 (2005)
Buckley, J.J.: Fuzzy hierarchical analysis. Fuzzy Set. Syst. **17**(3), 233–247 (1985)
Chan, F.T.S., Kumar, N.: Global supplier development considering risk factors using fuzzy extended AHP-based approach. Omega. Oxf. **35**(4), 417–431 (2007)
Chang, D.Y.: Application of the extent analysis method on fuzzy AHP. Eur. J. Oper. Res. **95**(3), 649–655 (1996)
Chen, J.K., Chen, I.S.: Improving the advantages of traditional manufacturing organizations in Taiwan based on vision using fuzzy AHP. Bus. Rev. Cambr. **10**(1), 82–88 (2008)
Dubois, D., Prade, H.: Fuzzy Sets and Systems: Theory and Applications. Academic Press, New York (1978)
Ensslin, L., Dutra, A., Ensslin, S.R.: MCDA: a constructivist approach to the management of human resources at a governmental agency. Int. Trans. Oper. Res. **7**(1), 79–100 (2000)
Fenton, N., Neil, M.: Making decisions: using Bayesian nets and MCDA. Knowl. Base. Syst. **14**(7), 307–325 (2001)
Guitouni, A., Martel, J.M.: Tentative guidelines to help choosing an appropriate MCDA method. Eur. J. Oper. Res. **109**(2), 501–521 (1998)
Hodgkin, J., Belton, V., Koulouri, A.: Supporting the intelligent MCDA user: A case study in multi-person multi-criteria decision support. Eur. J. Oper. Res. **160**(1), 172–189 (2005)
Kahraman, C., Kaya, I.: A fuzzy multicriteria methodology for selection among energy alternatives. Expert Syst. Appl. **37**(9), 6270–6281 (2010)
Kahraman, C., Cebeci, U., Ruan, D.: Multi-attribute comparison of catering service companies using fuzzy AHP: The case of Turkey. Int. J. Prod. Econ. **87**(2), 171–184 (2004)
Kahraman, C., Ertay, T., Büyüközkan, G.: A fuzzy optimization model for QFD planning process using analytic network approach. Eur. J. Oper. Res. **171**(2), 390–411 (2006)
Kannan, G.: Fuzzy approach for the selection of third party reverse logistics provider. Asia Pac. J. Mark. Logist. **21**(3), 397–416 (2009)
Körner, V., Zimmermann, H.D.: Management of customer relationship in business media (MCR-BM). Electron. Mark. **10**(3), 162–168 (2000)
Kuo, R.J., Chi, S.C., Kao, S.S.: A decision support system for selecting convenience store location through integration of fuzzy AHP and artificial neural network. Comput. Ind. **47**(2), 199–214 (2002)
Kwong, C.K., Bai, H.: Determining the importance weights for the customer requirements in QFD using a fuzzy AHP with an extent analysis approach. IIE Trans. **35**(7), 619–626 (2003)
Lee, K.L., Huang, W.C., Teng, J.Y.: Locating the competitive relation of global logistics hub using quantitative SWOT analytical method. Qual Quant. **43**, 87–107 (2009)
Li, J., Chen, S.: Evaluating the architectural design services by using Fuzzy AHP. Bus. Rev. Cambr. **13**(2), 129–137 (2009)
Murtaza, M.B.: Fuzzy-AHP application to country risk assessment. Am. Bus. Rev. **21**(2), 109–116 (2003)
Neves, L.P., Dias, L.C., Antunes, C.H., Martins, A.G.: Structuring an MCDA model using SSM: A case study in energy efficiency. Eur. J. Oper. Res. **199**(3), 834–845 (2009)
Perçin, S.: Use of fuzzy AHP for evaluating the benefits of information-sharing decisions in a supply chain. J. Enterp. Inf. Manag. **21**(3), 263–284 (2008)
Saaty, T.L.: The Analytic Hierarchy Process: Planning, Priority Setting, Resource Allocation. McGraw-Hill Companies Inc., New York (1980)
Sohn, K.Y., Yang, J.W., Kang, C.S.: Assimilation of public opinions in nuclear decision-making using risk perception. Ann. Nucl. Energ. **28**(6), 553–563 (2001)
Srdjevic, B., Medeiros, Y.D.P.: Fuzzy AHP assessment of water management plans. Water Resour. Manag. **22**(7), 877–894 (2008)
Tsai, H.Y., Chang, C.W., Lin, H.L.: Fuzzy hierarchy sensitive with Delphi method to evaluate hospital organization performance. Exp. Syst. Appl. **37**(8), 5533–5541 (2010)

Uppuluri, V.R.R.: Logarithmic least-squares approach to Saaty's decision problems. In: Lever, W.E., Shepherd, D.E., Ward, R.C., Wilson, D.G. (eds.) Mathematics and Statistics Research Department Progress Report, pp. 19–21. Oak Ridge National Laboratory, Oak Ridge (1978)
Uppuluri, V.R.R.: Expert Opinion and Ranking Methods. Oak Ridge National Laboratory, NRC FIN No. B044, Oak Ridge (1983)
Weck, M., Klocke, F., Schell, H., Rüenauver, E.: Evaluating alternative production cycles using the extended fuzzy AHP method. Eur. J. Oper. Res. **100**(2), 351–366 (1997)
Zadeh, L.A.: Fuzzy sets. Inf. Control. **8**(3), 338–353 (1965)
Zadeh, L.A.: The concept of a linguistic variable and its application to approximate reasoning Part I, II and III. Inf. Sci. **8**, 199–249, 301–357; **9**, 43–80 (1975)