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Effects of the optimal step toll scheme on equilibrium commuter behaviour

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This paper derives commuters' equilibrium queuing costs and equilibrium schedule delay costs before and after levying the optimal step tolls at a queuing bottleneck. Dealing with these equilibrium costs technically one can forecast some changes in equilibrium commuter behaviour from the no-toll to the optimal step toll cases. There is some useful information provided in this paper. First, the number of commuters who will or will not pay the tolls can be investigated before tolling a queuing bottleneck. Second, all commuters' departure time switching decisions from the no-toll to the tolled cases can be investigated before tolling. Third, the increased leisure time in the morning to the toll payer due to depart from home later than their original departure times in the no-toll case can be investigated before tolling. The above information of equilibrium commuter behaviour, which the related literature has failed to provide, is useful to policy-makers if the optimal step toll scheme is considered to be put into practice.

I. INTRODUCTION

Laih (1994) developed a flexible pricing mechanism including the optimal single- and multi-step tolls to relieve commuting queuing in the morning at a road bottleneck. Four years later, Singapore implemented the Electronic Road Pricing (ERP) System in East Coast Parkway (ECP). Take the passenger car for instance; ERP rates in ECP for three time periods of 7:30–8:00 a.m., 8:00–9:00 a.m. and 9:00–9:30 a.m. from April 1998 to March 1999 are \$1, \$2 and \$1, respectively. The Land Transport Authority (LTA) of Singapore calls this toll structure 'the shoulder pricing'. In fact, one may observe that the toll structure of the shoulder pricing completely matches the optimal double-step toll scheme. Thereafter, ERP rates for passenger cars in ECP and other expressways from April 1999 have become changeable and irregular to respond to the multiple changes in commuter behaviour. However, such commuter behaviour changes are complicated and very difficult to forecast, especially in urban areas, because there are many commuting alternatives, such as different modes and routes, that commuters can take to reach their workplaces.

Providing a method of forecasting commuter behaviour changes from the no-toll to the tolled cases is not only valuable in the road pricing theory, but is also helpful for decision-makers to predict the various advantages that will be brought about from the congestion toll scheme. Unfortunately, a paper of such a kind is very difficult to find in the related literatures. In order to make a contribution towards this methodology, this paper provides a methodological framework to forecast commuter behaviour changes from the original no-toll case to the optimal step toll case.

This paper first derives the complete and regular values in toll structures of the optimal n-step toll schemes (where $n = 1, 2, 3, \ldots$) that are not provided in the previous literature. These are the basis to develop the methodological framework used to forecast equilibrium commuter behaviour. Next, this paper derives the equilibrium queuing cost and equilibrium schedule delay cost to each commuter before and after implementing the optimal n-step toll schemes. These equilibrium costs are indispensable tools to predict commuter behaviour changes from the no-toll to the tolled cases. Dealing with these equilibrium costs technically, some important results can be obtained as fol-

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lows. First, the number of commuters who will pay the optimal step toll to cross a queuing bottleneck. Second, all commuters' departure time switching decisions from the no-toll to the tolled cases. Third, the length of the increased leisure time in the morning to the toll payer due to depart from home later than their original departure times in the no-toll case.

The paper is organized as follows. A concise review of the no-toll equilibrium, the optimal time-varying toll, and the basic optimal step toll structure for a queuing bottleneck model which have been discussed in the previous literature is given in section II. Methodological frameworks used to forecast equilibrium commuter behaviour changes from the no-toll to the optimal single- and multi-step toll cases are developed in sections III and IV, respectively. In addition, various information about commuter behaviour forecasting are provided based on the methodological framework. Finally, practical implications of the main results provided in this paper are addressed carefully in section V.

II. BACKGROUND REVIEWS

Queuing often develops in front of the entry to a road bottleneck during the commuting rust hour. The model of pricing a queuing bottleneck first developed by Vickrey (1969) and extended by Small (1982), De Palma and Arnott (1986), Cohen (1987), Newell (1987), Braid (1989), Arnott *et al.* (1990), Tabuchi (1993), Arnott *et al.* (1993), Laih (1994, 1998, 2000), Chen and Bernstein (1995), Yang and Lam (1996), Yang and Huang (1997), and Yang and Meng (1998). Among these researches, the optimal step toll scheme developed by Liah (1994) is picked up to examine various commuter behaviour changes from the no-toll to the tolled cases. There are two reasons for choosing this toll scheme. First, the optimal step toll scheme is derived from a pure queuing bottleneck model, i.e. commuters, one per car, have no alternative choices (including alternative modes and routes) but crossing a queuing bottleneck to reach their workplaces. This simple model makes the commuter behaviour prediction possible if the optimal toll is put into practice. Second, the optimal step toll scheme has regular variations in both toll levels and charging lengths as the number of pricing steps is increased (or decreased) one by one. These regularities, which other toll schemes do not have, make the variations in commuter behaviour predictable as the number of pricing steps changes.

The basis assumptions for a queuing bottleneck model are as follows. First, fixed number of commuters, one per care, must cross a road bottleneck to reach their workplaces. Second, these commuters are indifferent in time values, and they choose their departure times rationally based on the commuting cost minimization principle. Third, the total commuting cost for each commuter includes the queuing time cost, the schedule delay cost (the costs of arriving at work earlier or later than the work start time) and the toll (if any). For simplicity, both the queuing time cost and the schedule delay cost are usually assumed to be linear. Fourth, all commuters' commuting demands to the bottleneck are perfectly inelastic. Fifth, queuing only developed at the entry to the bottleneck due to the capacity reduction. Sixth, commuting time other than waiting in the queue due to the bottleneck is constant for departure time decisions. Therefore, one may consider that a commuter arrives at the entry to the bottleneck as soon as (s)he departs from home, and arrives at the workplace immediately when (s)he gets rid of the queuing.

Let us first review the no-toll equilibrium case. Under the sixth assumption mentioned above, there are three possible schedulings of arriving pattern at the workplace.

$$\text{If } (t + T_Q(t)) < t^* \text{ (time-early scheduling),}$$
$$\text{then } T_E(t) = t^* - (t + T_Q(t)) \text{ and } T_L(t) = 0 \quad (1.1)$$
$$\text{If } (t + T_Q(t)) = t^* \text{ (on-time scheduling),}$$
$$\text{then } T_E(t) = T_L(t) = 0 \quad (1.2)$$
$$\text{If } (t + T_Q(t)) > t^* \text{ (time-late scheduling),}$$
$$\text{then } T_L(t) = (t + T_Q(t)) - t^* \text{ and } T_E(t) = 0 \quad (1.3)$$

Figure 1 clearly illustrates the above situations. Both situations of (1.1) and (1.3) are often called schedule delay for morning commuters in the bottleneck queuing model. Note that definitions to all notations used in this paper are listed in the Appendix.

Now consider an unpriced bottleneck. According to the first assumption, the commuter's problem can be expressed as

$$\text{Minimize } TC(t) = \alpha T_Q(t) + \beta T_E(t) + \gamma T_L(t)$$
$$\text{where } t_q \le t \le t_{q'}. \quad (2)$$

Substituting Equations (1.1)–(1.3) into Equation (2), the total commuting cost, TC, can be expressed as

$$TC(t) = \alpha \cdot T_Q(t) + \beta \cdot [t^* - (t + T_Q(t))],$$
$$\text{for } t_q \le t + T_Q < t^* \quad (\text{or } t < \tilde{t}\,) \quad (3.1)$$
$$TC(t) = \alpha \cdot T_Q(t), \quad \text{for } t + T_Q = t^* \quad (\text{or } t = \tilde{t}\,) \quad (3.2)$$
$$TC(t) = \alpha \cdot T_Q(t) + \gamma[(t + T_Q(t)) - t^*],$$
$$\text{for } t^* < t + T_Q \le t_{q'} \quad (\text{or } t > \tilde{t}\,). \quad (3.3)$$

Equations (3.1), (3.2) and (3.3) are total commuting costs related to the time-early, on time, and time-late schedulings, respectively. Because all commuters seek to minimize the total commuting costs, a stable equilibrium can be reached when each commuter's total commuting cost is equal for all departure times. Accordingly, the equilibrium condition can be expressed as $d(TC)/dt = 0$ for all t.

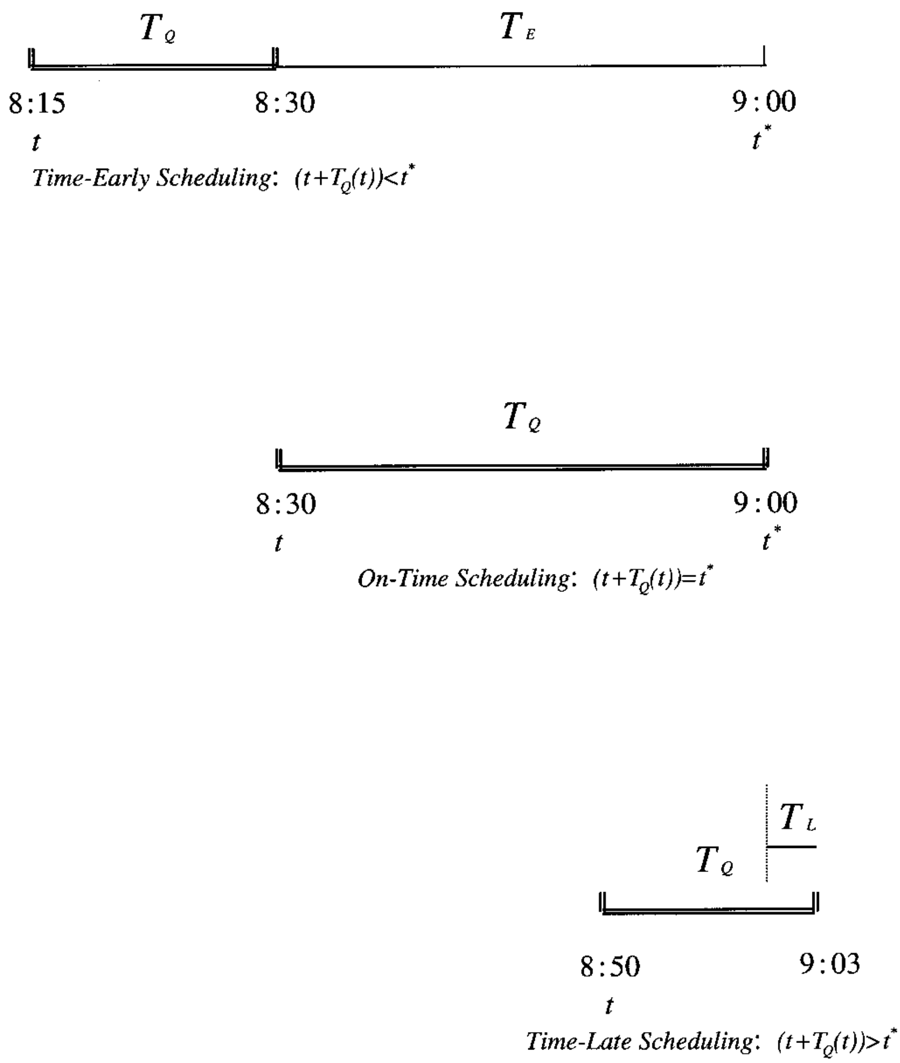

Fig. 1. *Three possible schedulings of arriving patterns at work*

Applying this rule to equations (3.1) and (3.3), the slopes of the queueing time can then be obtained as:

$$\mathrm{d}T_Q/\mathrm{d}t = \beta/(\alpha-\beta), \quad \text{for} \quad t_q \le t + T_Q < t^* \quad (\text{or } t < \tilde{t}\,) \tag{4.1}$$

$$\mathrm{d}T_Q/\mathrm{d}t = -\gamma/(\alpha+\gamma), \quad \text{for} \quad t^* < t + T_Q \le t_{q'} \quad (\text{or } t > \tilde{t}\,). \tag{4.2}$$

Equations (4.1) and (4.2) are positive and negative, respectively, because $\gamma > \alpha > \beta > 0$. Then it is clear that the equilibrium queueing time cost, $\alpha \cdot T_Q^e(t)$, increases linearly from t_q to a maximum value at $\tilde{t}$ during the time-early period, and then decreases linearly from $\tilde{t}$ to $t_{q'}$ during the time-late period. See the triangle $t_qMt_{q'}$ in Figures 2 and 3.

The decision we are now facing is how to locate $\tilde{t}$, t_q and $t_{q'}$ under the no-tall equilibrium. By using two equations, namely, $\tilde{t} = t^* - T_Q(\tilde{t}\,)$ and $N = s(t_{q'} - t_q)$, $\tilde{t}$, t_q and $t_{q'}$ can be obtained as

$$\tilde{t} = t^* - [\beta\gamma/\alpha(\beta+\gamma)](N/s) \tag{5.1}$$

$$t_q = t^* - [\gamma/(\beta+\gamma)](N/s)) \tag{5.2}$$

$$t_{q'} = t^* + [\beta/(\beta+\gamma)](N/s). \tag{5.3}$$

Since all commuters who depart during $[t_q, t_{q'}]$ have the same cost in equilibrium, by substituting $T_Q(\tilde{t}\,) = t^* - \tilde{t}$ into Equation (3.2), the equilibrium commuting cost per auto-commuter can then be expressed as

$$TC^e = [\beta\gamma/(\beta+\gamma)](N/s). \tag{6}$$

Next, let us consider the toll scheme to a bottleneck. The optimal time-varying toll is defined as a series of tolls that will completely eliminate the efficiency loss of queuing times without making commuters worse off than they would be in the no-toll equilibrium. In order to attain such an objective, it is then necessary to impose a series of tolls, $\tau(t)$, which results in $T_Q(t) = 0$ and $TC(t) = TC^e$ for

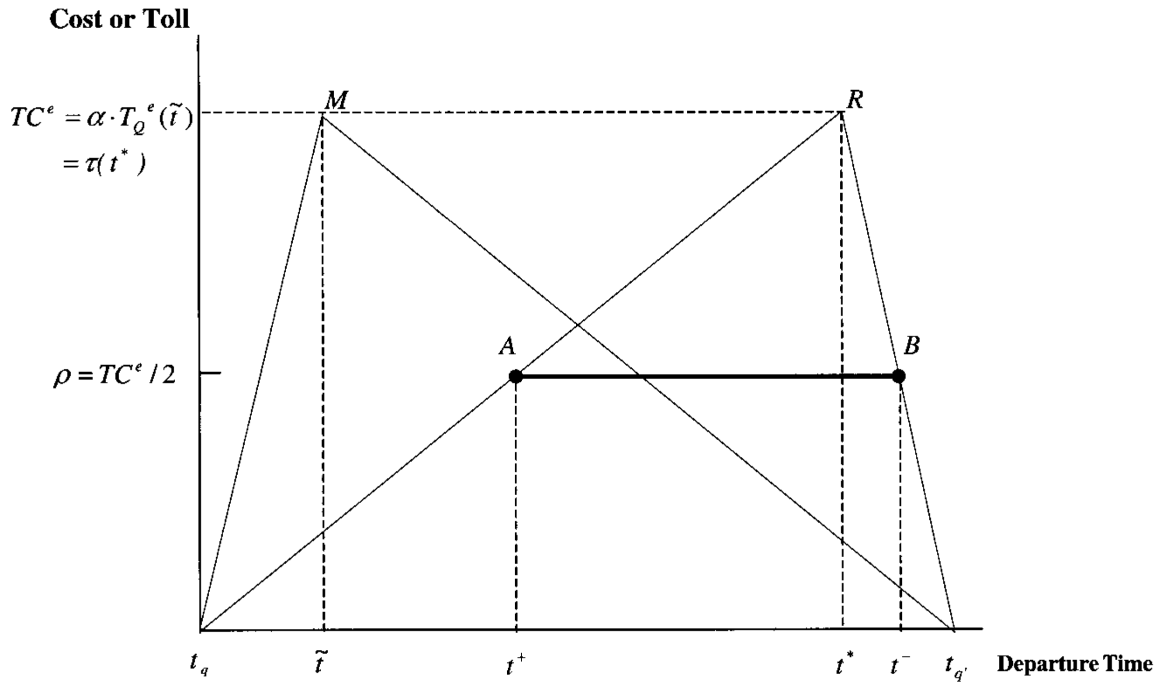

Fig. 2. *The optimal single-step toll structure* t^+ ABt$^-$

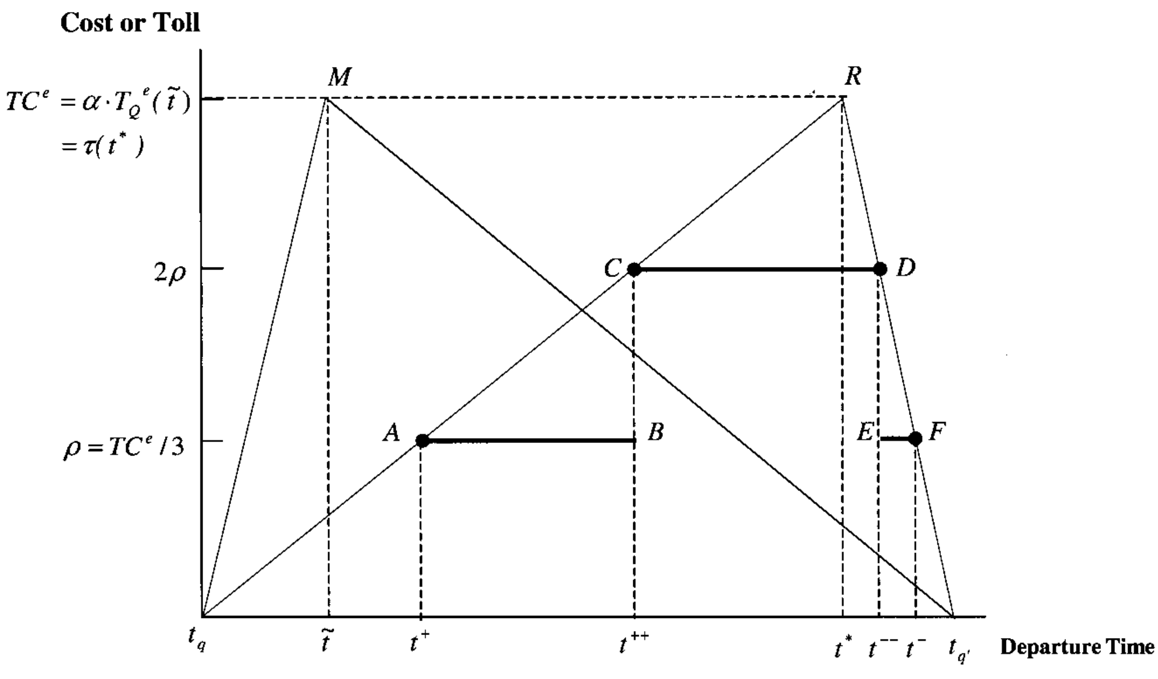

Fig. 3. *The optimal double-step toll structure* t^+ ABCDEFt$^-$

all t in Equations (3.1)–(3.3). Accordingly, $\tau(t)$ can be written as:

$$\tau(t) = TC^e - \beta(t^* - t), \quad \text{for } t_q \le t < t^* \tag{7.1}$$

$$\tau(t) = TC^e, \quad \text{for } t = t^* \tag{7.2}$$

$$\tau(t) = TC^e - \gamma(t - t^*), \quad \text{for } t^* < t \le t_{q'}. \tag{7.3}$$

The shape of the optimal time-varying toll scheme is triangular because of continuously changeable charges throughout the queuing period $[t_q, t_{q'}]$. See triangle $t_q R t_{q'}$ in Figures 2 and 3. The slope for Equation (7.1) during $[t_q, t^*)$ is always smaller than the shape for Equation (7.3) during $[t_q, t^*)$ since $\beta < \gamma$. The maximum optimal time-varying toll is located at the work start time, i.e. $\tau(t^*)$.

This is reasonable because commuters are willing to pay the highest optimal time-varying toll to arrive at work on time without incurring any schedule delay costs.

The optimal time-varying toll is capable of eliminating queuing time completely, but has practical difficulties because it requires continuously changeable charges. Due to these difficulties, a step toll scheme has been considered as an alternative to reduce queuing time. The single- and multi-step tolls inscribed in the optimal fine toll triangle are first developed by Laih (1994) to reduce the queuing time to a described level. As shown in Figures 2 and 3, the single- and double-step toll schemes are shaped as $t^{+}ABt^{-}$ and $t^{+}ABCDEFt^{-}$, respectively. A step toll structure with the maximum toll revenue inscribed in the optimal time-varying toll triangle is defined as the optimal step toll scheme to remove the largest proportion of the total queuing time (see, for example, Figure 2 to annotate this definition). If point A is moved to the left side horizontally and located on, say $\tilde{t}$, then the total commuting cost to any one who departs during this extended tolled period $[\tilde{t}, t^{+})$ will be larger than his original no-toll equilibrium commuting cost, TC^{e}. This is because he is overtolled from $\tau(t)$, where $t \in [\tilde{t}, t^{+})$, up to ρ. Hence only when $\overline{AB}$ inscribed in $\Delta t_{q}Rt_{q'}$ can the maximum toll revenue be obtained with commuters no worse off than they would be in the no-toll equilibrium. According to this definition to the optimal step toll scheme, the optimal single- and double-step tolls, as shown in Figures 2 and 3, divide the maximum optimal time-varying toll $\tau(t^{*})$ (or the equilibrium commuting cost TC^{e}) into two and three equal amounts, respectively. Laih (1994) has proved the existence of these structural characteristics.

Moreover, the effects of the optimal single- and double-step tolls on queuing reduction have been derived to be 1/2 and 2/3, respectively, of the total queuing time that existed in the no-toll equilibrium. Laih (1994) obtained these queuing reduction effects simply because the maximum toll revenues from the optimal single- and double-step toll schemes are 1/2 and 2/3, respectively, of the total equilibrium queuing costs in the no-toll case. However, Laih's work provided no information about commuter behaviour changes from the no-toll to the tolled cases that lead to a reduction in queuing at a road bottleneck. These problems will be dealt with in the following sections.

III. COMMUTER BEHAVIOUR IN THE OPTIMAL SINGLE-STEP TOLL CASE

This section derives the equilibrium queuing costs and equilibrium schedule delay costs under the optimal single-step toll scheme. By comparing the equilibrium queuing costs before and after tolling the bottleneck, the differences in distribution of departure rates throughout the queuing period can be known. Meanwhile, the size of departures for all types of commuters under the optimal single-step toll scheme can be obtained. Next, the equilibrium schedule delay costs before and after tolling the bottleneck will be compared to investigate all commuters' departure time switching decisions.

Equilibrium queuing costs and departure rates

The optimal single-step toll ρ ($=TC^{e}/2$), inscribed within the optimal fine toll $t_{q}Rt_{q'}$ in Figure 4, is applied at t^{+} and lifted at t^{-}. On the other hand, $t_{q}Mt_{q'}$ in the left side illustrates the no-toll equilibrium queuing cost. The slopes of $\overline{t_{q}M}$, $\overline{Mt_{q'}}$, $\overline{t_{q}R}$ and $\overline{Rt_{q'}}$ can be obtained as $\alpha\beta/(\alpha-\beta)$, $-\alpha\gamma/(\alpha+\gamma)$, β and $-\gamma$, respectively, by using values of t_{q}, $\tilde{t}$, t^{*}, $t_{q'}$ and TC^{e} that we have derived in section II.

In Figure 4, t' and $t^{\#}$ are two important time spots that need to be discussed. Let us first interpret t'. Because a bottleneck is fully utilized throughout the queuing period $[t_{q}, t_{q'}]$, the last person who will not pay the toll before t^{+} arrives at his workplace just before the first person who will pay the toll at t^{+}. This means that each has almost the same time-early costs. Because the queuing cost to the latter is zero, and also because both have the same commuting cost in equilibrium, the former must incur a queuing cost that is equal to the amount to the toll ρ, and consequently must depart ρ/α earlier. Hence there are no departures during the period $[t', t^{+})$, and the length of this time period is equal to ρ/α.

The implication for $t^{\#}$ is similar to t'. The last person who will pay the toll before t^{-} arrives at the workplace just before the first person who will not pay the toll when the toll is lifted. This means that each has almost the same time-late costs. Because both have the same commuting cost, and also because the queuing cost to the former is zero, the later must incur a queuing cost that is ρ higher than the former. This is impossible unless the latter has queued for a period of ρ/α before t^{-}. Therefore, we may consider that there is a mass of departing commuters waiting at the roadside in front of the tollgate entry to the bottleneck from $t^{\#}$ until t^{-}. They prepare to cross the bottleneck free once the toll is lifted or t^{-}. Consequently, the length of the time period $[t^{\#}, t^{-})$ is also equal to ρ/α.

Values of the toll and departure times appearing in Figure 4 are listed in Table 1. Note that t_{q} is now assumed to locate on the origin (i.e. $t_{q}=0$) in Figure 4 for the purpose of simplifying all values listed in Table 1. Therefore the values of $\tilde{t}$, t_{q} and $t_{q'}$ listed in Table 1 are $(t^{*}-(\gamma/(\beta+\gamma)(N/s))$ earlier than those that first appeared in Equations (5.1)–(5.3). Detailed computations to Table 1 are introduced as follows:

$$\rho = \frac{TC^{e}}{2} = \frac{\beta\gamma}{2(\beta+\gamma)}\left(\frac{N}{s}\right), \quad t_{q'} = \frac{N}{s} - t_{q} = \frac{N}{s},$$

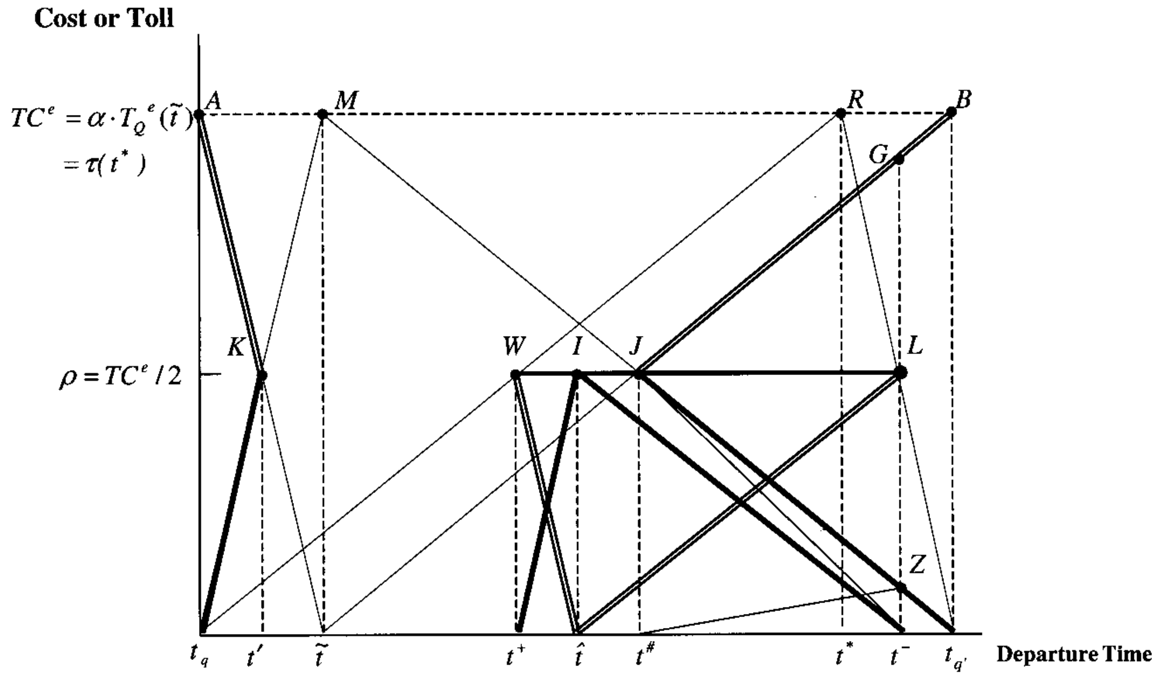

Fig. 4. *Equilibrium queuing costs and equilibrium schedule delay costs in the no-toll and optimal single-step toll cases*

Table 1. *Values of the optimal single-step toll and departure times*

ρ	t_q	$t_{q'}$	$\tilde{t}$	t^*	$\hat{t}$	t^+	t'	t^-	$t^{\#}$
$\frac{\beta\gamma}{2(\beta+\gamma)}\left(\frac{N}{s}\right)$	0	$\frac{N}{s}$	$\frac{\gamma(\alpha-\beta)}{\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{\gamma}{\beta+\gamma}\left(\frac{N}{s}\right)$	$\frac{\gamma(2\alpha-\beta)}{2\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{\gamma}{2(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{\gamma(\alpha-\beta)}{2\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{\beta+2\gamma}{2(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{2\alpha\gamma+\alpha\beta-\beta\gamma}{2\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$

$$\tilde{t} = t_q + \overline{t_q\tilde{t}} = t_q + \frac{TC^e}{\alpha\beta/(\alpha-\beta)} = \frac{\gamma(\alpha-\beta)}{\alpha(\beta+\gamma)}\left(\frac{N}{s}\right),$$

$$t^* = \tilde{t} + T_Q(\tilde{t}) = \tilde{t} + \frac{TC^e}{\alpha} = \frac{\gamma}{\beta+\gamma}\left(\frac{N}{s}\right),$$

$$\hat{t} = t^* - T_Q(\hat{t}) = t^* - \frac{\rho}{\alpha} = \frac{\gamma(2\alpha-\beta)}{2\alpha(\beta+\gamma)}\left(\frac{N}{s}\right),$$

$$t^+ = t_q + \overline{t_q t^+} = t_q + \frac{\rho}{\beta} = \frac{\gamma}{2(\beta+\gamma)}\left(\frac{N}{s}\right),$$

$$t' = t^+ - \overline{t't^+} = t^+ - \frac{\rho}{\alpha} = \frac{\gamma(\alpha-\beta)}{2\alpha(\beta+\gamma)}\left(\frac{N}{s}\right),$$

$$t^- = t_{q'} - \overline{t^- t_{q'}} = t_{q'} - \frac{\rho}{\gamma} = \frac{\beta+2\gamma}{2(\beta+\gamma)}\left(\frac{N}{s}\right),$$

$$t^{\#} = t^- - \overline{t^{\#}t^-} = t^- - \frac{\rho}{\alpha} = \frac{2\alpha\gamma+\alpha\beta-\beta\gamma}{2\alpha(\beta+\gamma)}\left(\frac{N}{s}\right).$$

Table 2 illustrates equilibrium results for all departure intervals under the optimal single-step toll scheme. Note that there exists a blanket departure time interval $[t', t^+)$ because nobody departs during this time period that we have mentioned before. Commuters of groups B, C and D depart during the tolled period $[t^+, t^-)$. Except for group D, which escapes from being tolled as we have mentioned before, groups B and C depart during the tolled periods and pay the toll to cross the bottleneck. Groups A and E do not need to pay the toll because they depart during the no-toll periods. Moreover, only groups A and B will arrive at their workplaces earlier than the work start time because they depart before $\hat{t}$. These results are arranged as columns (1)–(3) in Table 2.

Since equilibrium will be achieved as long as all commuters have the same commuting costs throughout the queuing period, there are two kinds of equilibrium conditions for the early and later arrivals. One is $TC(t) = TC(t_q)$ for groups A and B of the early arrival, and the other is $TC(t) = TC(t_{q'})$ for groups C, D and E of the late arrival. These equilibrium conditions then can be expressed as follows:

$$\alpha \cdot T_Q(t) + \beta[t^* - (t + T_Q(t))] = \beta \cdot t^*, \quad \text{for } t_q \le t \le t' \tag{8.1}$$

Table 2. *Equilibrium results under the optimal single-step toll scheme*

(1) Groups	(2) Departure time intervals	(3) Types of commuters	(4) EQC: $\alpha \cdot T_Q^e(t)$	(5) EDR	(6) Number of commuters	(7) ESDC: $\beta \cdot T_E^e(t)$ or $\gamma \cdot T_L^e(t)$
A	$t_q \le t < t'$ (no-toll period)	(a) Toll free (b) Early arrivals	$\frac{\alpha\beta \cdot t}{\alpha - \beta}$	$\frac{\alpha \cdot s}{\alpha - \beta}$	$\frac{\gamma \cdot N}{2(\beta + \gamma)}$	$\frac{-\alpha\beta \cdot t}{\alpha - \beta} + \frac{\beta\gamma}{(\beta + \gamma)}\left(\frac{N}{s}\right)$
	$t' \le t < t^+$ (no-toll period)	None	0	0	0	0
B	$t^+ \le t < \hat{t}$ (tolled period)	(a) Toll payers (b) Early arrivals	$\frac{\alpha\beta \cdot t}{\alpha - \beta} - \frac{\alpha\beta\gamma}{2(\beta + \gamma)(\alpha - \beta)}\left(\frac{N}{s}\right)$	$\frac{\alpha \cdot s}{\alpha - \beta}$	$\frac{\gamma \cdot N}{2(\beta + \gamma)}$	$\frac{-\alpha\beta \cdot t}{\alpha - \beta} + \frac{\beta\gamma(2\alpha - \beta)}{2(\beta + \gamma)(\alpha - \beta)}\left(\frac{N}{s}\right)$
C	$\hat{t} < t < t^-$ (tolled period)	(a) Toll payers (b) Late arrivals	$\frac{-\alpha\gamma \cdot t}{\alpha + \gamma} + \frac{\alpha\gamma(\beta + 2\gamma)}{2(\beta + \gamma)(\alpha + \gamma)}\left(\frac{N}{s}\right)$	$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\beta \cdot N}{2(\beta + \gamma)}$	$\frac{\alpha\gamma \cdot t}{\alpha + \gamma} - \frac{\gamma^2(2\alpha - \beta)}{2(\beta + \gamma)(\alpha + \gamma)}\left(\frac{N}{s}\right)$
D	$t^\# \le t < t^-$ (tolled period)	(a) Toll free (b) Late arrivals	$\frac{-\alpha\gamma \cdot t}{\alpha + \gamma} + \frac{\alpha\gamma}{\alpha + \gamma}\left(\frac{N}{s}\right)$	$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\beta\gamma \cdot N}{2(\alpha + \gamma)(\beta + \gamma)}$	$\frac{\alpha\gamma \cdot t}{\alpha + \gamma} - \frac{\gamma^2(\alpha - \beta)}{(\beta + \gamma)(\alpha + \gamma)}\left(\frac{N}{s}\right)$
E	$t^- \le t \le t_{q'}$	(a) Toll free (b) Late arrivals	$\frac{-\alpha\gamma \cdot t}{\alpha + \gamma} + \frac{a\gamma}{\alpha + \gamma}\left(\frac{N}{s}\right)$	$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\alpha\beta \cdot N}{2(\alpha + \gamma)(\beta + \gamma)}$	$\frac{\alpha\gamma \cdot t}{\alpha + \gamma} - \frac{\gamma^2(\alpha - \beta)}{(\beta + \gamma)(\alpha + \gamma)}\left(\frac{N}{s}\right)$

$$\alpha \cdot T_Q(t) + \beta[t^* - (t + T_Q(t))] + \rho = \beta \cdot t^*, \quad \text{for } t^+ \le t < \hat{t} \tag{8.2}$$

$$\alpha \cdot T_Q(t) + \gamma[(t + T_Q(t)) - t^*] + \rho = \gamma(t_{q'} - t^*), \quad \text{for } \hat{t} < t < t^- \tag{8.3}$$

$$\alpha \cdot T_Q(t) + \gamma[(t + T_Q(t)) - t^*] = \gamma(t_{q'} - t^*), \quad \text{for } t^\# \le t < t^- \text{ or } t^- \le t \le t_{q'}. \tag{8.4}$$

The above equilibrium conditions (8.1)–(8.4) are established for groups A, B, C and D (or E), respectively, where $[t^*\text{-}(t+T_Q(t))]$ and $[(t+T_Q(t))\text{-}t^*]$ indicate the time periods for commuters arriving at work early (T_E) and late (T_L), respectively. Besides, the values of $TC(t_q)$ and $TC(t_{q'})$ are $\beta{\cdot}t^*$ and $\gamma(t_{q'} - t^*)$, respectively, because $t_q = 0$ and $T^Q(t_q) = T_Q(t_{q'}) = 0$.

Equilibrium queuing costs ($EQC : \alpha \cdot T^e_Q(t)$), for groups A–E, listed in column (4) of Table 2 are obtained based on Equations (8.1)–(8.4). As shown in Figure 4, the equilibrium queuing costs for groups A–E under the optimal single-step toll scheme are thick lines $\overline{t_qK}$, $\overline{t^+I}$, $\overline{It^-}$, $\overline{JZ}$ and $\overline{Zt_{q'}}$, respectively. The slope of $\overline{t_qK}$ and $\overline{t^+I}$ for all early arrivals is $\alpha\beta/(\alpha-\beta)$, which is the same as the slope of the equilibrium queuing cost ($\overline{t_qM}$) to all early arrivals in the no-toll case. Note that there are no thick lines of equilibrium queuing costs through the departure period $[t', t^+)$ since no-one departs during this period. In addition, the length of the queue will be reduced to zero at t^+ because of $T_Q(t') = \rho/\alpha = t^+ - t'$. On the other hand, the slope of $\overline{It^-}$, $\overline{JZ}$ and $\overline{Zt_{q'}}$, for all late arrivals is $-\alpha\gamma/(\alpha+\gamma)$, which is the same as the slope of the equilibrium queuing cost ($\overline{Mt_{q'}}$) to all late arrivals in the no-toll case. Note the group D's equilibrium queuing costs incurred before and after t^- in Figure 4 are $\overline{Jt^-}$ and $\overline{t^\#Z}$, respectively. The former is the decreasing equilibrium queuing costs, from ρ to zero, to those commuters avoid crossing the bottleneck during the tolled period $[t^\#, t^-)$. The latter is the increasing equilibrium queuing costs, from zero to $\alpha{\cdot}\rho/(\alpha+\gamma)$, to the same commuters that cross the bottleneck for free after t^-. Consequently, the total equilibrium queuing cost for group D is $\overline{JZ}$ $(= \overline{Jt^-} + \overline{t^\#Z})$.

The equilibrium departure rates (EDR) for groups A–E in column (5) of Table 2 are obtained by using the corresponding equilibrium queuing time ($T^e_Q(t)$). Figure 5 shows the EDR distributions for both the no-toll and the optimal single-step toll cases. The two cases are shown as the dotted line area and the shadowed area, respectively.

In the no-toll case, the equilibrium queuing costs ($\alpha \cdot T^e_Q(t)$) for all early and late arrivals in Figure 4 are $\overline{t_qM}$ and $\overline{Mt_{q'}}$, respectively, and the slopes for the former and latter are $\alpha\beta/(\alpha-\beta)$ and $-\alpha\gamma/(\alpha+\gamma)$, respectively. Accordingly, the marginal departure rate ($= \mathrm{d}(s \cdot T^e_Q(t))/(\mathrm{d}t)$) for the early and late arrivals are $\beta s/(\alpha-\beta)$ and $-\gamma s/(\alpha+\gamma)$, respectively. Since the bottleneck is fully utilized through the queuing period, EDR for the early and late arrivals in the no-toll case are therefore equal to $\alpha s/(\alpha-\beta)(= s + (\beta s/(\alpha-\beta))$ and $\alpha s/(\alpha+\gamma)(= s - (\gamma s/(\alpha+\gamma))$, respectively. See the dotted line area in Figure 5.

The EDR for the optimal single-step toll case is somewhat more complicated than the no-toll case. First, because $\overline{t_qK}$ and $\overline{t_qM}$ in Figure 4 coincide during $[t_q, t')$, the EDR for this departure period is the same as that in the no-toll case. Second, since there are no departures during $[t', t^+)$, the EDR is zero for this departure period. Third, because the slopes of $\overline{t^+I}$ and $\overline{It^-}$ in Figure 4 are the same as the slope of $\overline{t_qM}$ and $\overline{Mt_{q'}}$, respectively, the EDR for the two periods $[t^+, \hat{t})$ and $[\hat{t}, t^-)$ are $\alpha s/(\alpha-\beta)$ and $\alpha s/(\alpha+\gamma)$, respectively. Note that there exist group D commuters who depart during the tolled period $[t^\#, t^-)$ but decide to cross the bottleneck for free after t^-. It is clear that their departure time period $[t^\#, t^-)$ overlaps with group C. Because their marginal departure rate is equal to $\alpha s/(\alpha+\gamma)$ (i.e. the slope of $\overline{t^\#Z}$ equals $\alpha^2/(\alpha+\gamma)$, the total EDR for the departure period $[t^\#, t^-)$ in Figure 5 therefore becomes $2\alpha s/(\alpha+\gamma)$. Finally, $\overline{Zt_{q'}}$ and $\overline{Mt_{q'}}$ in Figure 4 coincide during $[t^-, t_{q'}]$, so the EDR for this departure period is also $\alpha s/(\alpha+\gamma)$.

The numbers of commuters listed in column (6) of Table 2 are computed by multiplying the lengths of departure intervals and corresponding values of EDR together. The former can be obtained by using the related departure time values listed in Table 1, and the latter are already shown in column (5). From column (6), it is clear that the numbers of the early and late arrivals are equal to $\frac{\gamma \cdot N}{\beta+\gamma}$ and $\frac{\beta \cdot N}{\beta+\gamma}$ respectively, and both are independent of α.

Equilibrium departure time switching decisions

The previous discussion has shown the distribution differences in the equilibrium departure rate before and after pricing a queuing bottleneck with the optimal single-stop toll. However, these results give no indication of the commuters' departure time switching decisions from the no-toll to the tolled cases. These problems will be solved by comparing the equilibrium schedule delay costs between the two cases. The reason why we will make such comparisons is illustrated as follows. Because the congestion toll derived from our model is simply the money cost to the toll payer required to save the same amount of queuing costs, the equilibrium schedule delay cost (i.e. either the early cost, $\beta \cdot T^e_E(t)$ or the late cost, $\gamma \cdot T^e_L(t)$) in the optimal step toll case must be the same as that in the original no-toll case to maintain the equilibrium commuting cost (TC^e). For this purpose, all commuters with the same values of β and γ will not alter their original preferred arrival times at work in the no-toll case if the bottleneck is tolled. All commuters' equilibrium departure time switching decisions then can be investigated by this principle, and we call it 'the invariant schedule delay costs principle'.

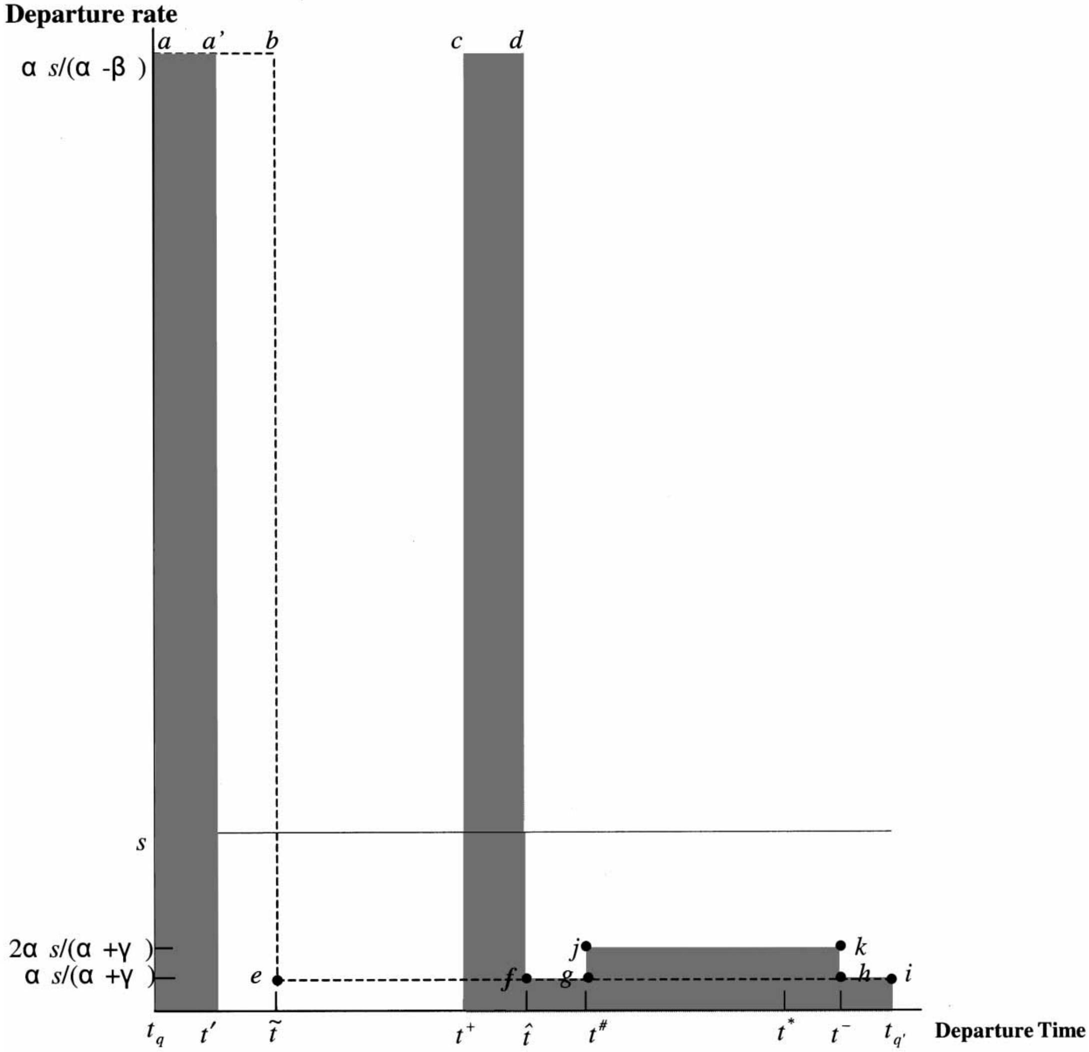

Fig. 5. *Equilibrium departure rates in the no-toll and optimal single-step toll cases*

Let us first analyse the equilibrium schedule delay cost (ESDC) under the optimal single-step toll case. ESDC to groups A and B is the equilibrium early cost $\beta \cdot T_E^e(t)$. On the other hand, ESDC to other groups C, D and E is the equilibrium late cost $\gamma \cdot T_L^e(t)$. Therefore, the contents of the equilibrium commuting cost (TC^e) to groups A–E under the optimal single-step toll scheme can be shown as $\alpha \cdot T_Q^e(t) + \beta \cdot T_E^e(t)$, $\alpha \cdot T_Q^e(t) + \beta \cdot T_E^e(t) + \rho$, $\alpha \cdot T_Q^e(t) + \gamma \cdot T_L^e(t) + \rho$, $\alpha \cdot T_Q^e(t) + \gamma \cdot T_L^e(t)$ and $\alpha \cdot T_Q^e(t) + \gamma \cdot T_L^e(t)$, respectively. Since the results of equilbrium queuing costs (EQC) for all departure intervals under the optimal single-step toll scheme have been shown in column (4) of Table 2, the corresponding values of ESDC required to achieve the equilibrium commuting cost $TC^e = (\beta\gamma/(\beta+\gamma)(N/s))$ can be easily obtained as shown in column (7). ESDC for groups A–E under the optimal single-step toll scheme are drawn as the doubled lines $\overline{\overline{AK}}$, $\overline{\overline{W\hat{t}}}$, $\overline{\overline{\hat{t}L}}$, $\overline{\overline{JG}}$ and $\overline{\overline{GB}}$, respectively in Figure 4. The slope of $\overline{\overline{AK}}$ and $\overline{\overline{W\hat{t}}}$ for all early arrivals is identical and equal to $-\alpha\beta/(\alpha-\beta)$. This is the same as the slope of $\overline{A\hat{t}}$ which represents all early arrivals' ESDC in the no-toll case. On the other hand, the slope of $\overline{\overline{\hat{t}L}}$, $\overline{\overline{JG}}$ and $\overline{\overline{GB}}$ for all late arrivals is identical and equal to $\alpha\gamma/(\alpha+\gamma)$. This is also the same as the slope of $\overline{\tilde{t}B}$, which represents all lat arrivals' ESDC in the no-toll case.

Next, the detailed discussion of all commuters' departure time switching decisions will be made by 'the invariant schedule delay costs principle' that we have mentioned above. First, group A commuters will not alter their original departure times in the no-toll case when the bottleneck is priced with the optimal single-step toll. This is because the equilibrium early costs $\overline{\overline{AK}}$ for both the no-toll and optimal single-step toll cases coincide during the departure period $(t_{q'}, t')$. The part of $t_q t' d' a$ in Figure 5 represents those commuters of group A. Second, because the equilibrium early cost $(\overline{\overline{W\hat{t}}})$ in the optimal single-step toll case and the equilibrium early cost $(\overline{K\tilde{t}})$ in the no-toll case are two identical and parallel lines, all commuters of group B that originally depart during the period $(t', \tilde{t})$ in the no-toll case will shift their departures to the period $(t^+, \hat{t})$ in the optimal single-step toll case. Therefore, the dotted line area $t'\tilde{t}ba'$ in Figure 5 will move to the shadowed part of $t^+\hat{t}dc$. Similarly, because the equilibrium late cost $(\hat{\hat{t}L})$ in the optimal single-step toll case and the equilibrium late cost $(\tilde{t}J)$ in the no-toll case are two identical and parallel lines, all commuters of group C that originally depart during the period $[\tilde{t}, t^{\#}]$ in the no-toll case will shift their departures to the period $[\hat{t}, t^-]$ in the optimal single-step toll case. Therefore, the dotted line area $\tilde{t}t^{\#}ge$ in Figure 5 will move to the shadowed part of $\hat{t}t^-hf$. Third, because the equilibrium late costs $\overline{\overline{JG}}$ in the optimal single-step toll case coincide with the equilibrium late cost $(\overline{\tilde{t}B})$ in the no-toll case, commuters in group D will not alter their original departure times in the no-toll case if the bottleneck is priced with the optimal single-step toll. Since the first floor of shadowed part during $[t^{\#}, t^-]$ in Figure 5 has been occupied by group C already, the shadowed part of *ghkj* in the second floor indicates commuters of group D. This supports the conclusion that the total equilibrium departure rate for the period $[t^{\#}, t^-]$ in the optimal single-step toll case is $2\alpha s/(\alpha+\gamma)$, which we made previously. Finally, because $\overline{\overline{GB}}$ also coincides with $\overline{\tilde{t}B}$, group E will not alter heir original departure times in the no-toll case if the bottleneck is priced with the optimal single-step toll. Therefore the shadowed part of $t^-t_{q'}ih$ in Figure 5 indicates commuters of group E.

The above outcomes are arranged and listed in Table 3. It is clear that commuters who choose the same departure times as they did in the original no-toll case are not the toll payers in the tolled case. These commuters are shown as Type I of early arrivals as well as Type IV of late arrivals in Table 3. The other part of commuters, i.e. Type II of early arrivals and type III of late arrivals, who alter their original departure times, are the toll payers.

IV. COMMUTER BEHAVIOUR IN THE OPTIMAL MULTI-STEP TOLL CASES

This section demonstrates the regularities in values of departure times, queuing costs, schedule delay costs, departure rates and in patterns of departure time shifts under the optimal n-step toll schemes (where $n = 1, 2, 3, \ldots, n$). In order to help readers to derive these regular results, Tables 4–11 and Figures 6–9 are provided to extend Tables 1–3 and Figures 4–5, respectively, which we have discussed in section III. It can be observed easily that structural characteristics in Figures 4 and 5 regularly appear in Figures 6–9. Then one may consider that all equilibrium outcomes derived from the optimal single-, double- and triple-step toll schemes must have some kind of regular relationship among them. If these regularities are confirmed, then all equilibrium results under the optimal n-step toll schemes (where $n = 1, 2, 3, \ldots$) can be obtained. The purpose of this section is to obtain these regular results and develop some enlightened propositions for policy-makers' references.

The methods to derive departure time values, various equilibrium costs and departure time switching decisions for the optimal multi-step toll cases are similar to the optimal single-step toll case that we have mentioned in section III. This section shows only these equilibrium results without providing detailed derivations.

Regular departure time values

Table 4 shows the departure time values under the optimal single- and multi-step toll schemes. Because t_q, $t_{q'}$, $\tilde{t}$ and t^* are four fixed time spots in our model, these values are independent of the number of pricing steps (n). It can be observed easily that other time spots have their own regularities in values from the optimal 1-step to 3-step toll schemes. By following these regularities, a series of regular departure time values under the optimal n-step toll schemes, as shown in Table 4, can be obtained. The blanket parts in this Table mean that the time spots are not existent in the corresponding step toll schemes.

Regular equilibrium results

Equilibrium conditions, equilibrium commuting costs (including equilibrium queuing costs, i.e. EQC, equilibrium schedule delay costs, i.e. ESDC, and tolls), equilibrium departure rates (EDR) and the numbers of commuters for both the time-early and time-late cases are shown as Tables 5–10. In these Tables, the regularities in equilibrium results under the optimal n-step toll schemes are obtained by following regular changes in equilibrium results from the optimal 1-step to 3-step toll schemes. Tables 5–10 are

Table 3. *Departure time switching decisions from the no-toll to the optimal single-step toll cases*

	Switch or not	Toll payers or not	Early or late arrivals at work
Type I commuters	No, keep staying at $[t_q, t']$	No	Early
Type I commuters	Yes, $[t', \tilde{t}\,) \to [t^+, \hat{t}\,)$	Yes	Early
Type III commuters	Yes, $[\tilde{t}, t^{\#}\,) \to [\hat{t}, t^-)$	Yes	Late
Type IV commuters	No, keep staying at $[t^{\#}, t_{q'}]$	No	Late

separated as two categories of the early and later arrival cases, hence $\hat{t}$ is the departure time that connects the two categories. Also, the blanket parts in these Tables show that the departure time intervals are not existent in the corresponding step toll schemes.

Tables 5 and 6 show different equilibrium conditions including $\alpha \cdot T_Q(t) + \beta[t^* - (t + T_Q(t))] = \beta \cdot t^*$, $\alpha \cdot T_Q(t) + \beta[t^* - (t - T_Q(t))] + n\rho = \beta \cdot t^*$, $\alpha \cdot T_Q(t) + \gamma[(t + T_Q(t)) - t^*] + n\rho = \gamma(t_{q'} - t^*)$ and $\alpha \cdot T_Q(t) + \gamma[(t + T_Q(t)) - t^*] = \gamma(t_{q'} - t^*)$ to all commuters that arrive at their workplaces either earlier or later than the work start time under the optimal n-step toll schemes (where $n = 1, 2, 3, \ldots, n$). According to these equilibrium conditions, one can obtain the equilibrium queuing costs, EQC: $\alpha \cdot T^e_Q(t)$. Then equilibrium schedule delay costs, ESDC: $\beta \cdot T^e_E(t)$ or $\gamma \cdot T^e_L(t)$, can be derived to maintain the equilibrium commuting cost TC^e for all t. All values of the equilibrium queuing cost, schedule delay cost and step tolls to all early and late arrivals under the optimal n-step toll schemes are shown in Tables 7 and 8. Finally, using $s + (\mathrm{d}(s \cdot T^e_Q(t))/(\mathrm{d}t))$ that we have mentioned in section III, we obtain the equilibrium departure rates (EDR) for each departure period. According to these results, we are able to compute the corresponding number of commuters. These results are shown in Tables 9 and 10.

Based on Tables 9 and 10, Proposition A, concerning the investigation of how many commuters will or will not pay the toll to cross a queuing bottleneck, is developed as follows.

PROPOSITION A: $n/(n+1)$ of all commuters (N) will pay the toll under the optimal n-step toll schemes. The number of toll payers who arrive at work early (including on-time) or late is $n \cdot \gamma \cdot N/((n+1)(\beta+\gamma))$ or $n \cdot \beta \cdot N/((n+1)(\beta+\gamma))$, respectively. On the other hand, the remaining $1/(n+1)$ of all commuters will not pay the toll to cross a queuing bottleneck. The number of these commuters who arrive at work early or late is $\gamma \cdot N/((n+1)(\beta+\gamma))$ or $\beta \cdot N/((n+1)(\beta+\gamma))$, respectively.

Proof:

(1) *Because the number of commuters who will pay the same level (step) of toll and arrive at work early (including on time) under the optimal n-step toll schemes in Table 9 is identical and equal to $\gamma \cdot N/((n+1)(\beta+\gamma))$, the total number of commuters who pay the tolls and depart through the following intervals $[t^+, t''), [t^{++}, t'''), \cdots, [t^{\overset{n}{+\cdots}}, \hat{t})$ is $\frac{n \cdot \gamma \cdot N}{(n+1)(\beta+\gamma)}$. On the other hand, because the number of commuters who pay the same level (step) of toll and arrive at work late under the optimal n-step toll schemes in Table 10 is also identical and equal to $\beta \cdot N/((n+1)(\beta+\gamma))$, the total number of commuters who pay the tolls and depart through the following intervals $(\hat{t}, t^{\overset{n}{-\cdots}})$, $(t^{\#}, t^{\overset{n-1}{-\cdots}}) = (t^{\#}, t^{\overset{n}{-\cdots}}) + (t^{\overset{n}{-\cdots}}, t^{\overset{n-1}{-\cdots}})$, $(t^{\#\#}, t^{\overset{n-2}{-\cdots}}) = (t^{\#\#}, t^{\overset{n-1}{-\cdots}}) + (t^{\overset{n-1}{-\cdots}}, t^{\overset{n-2}{-\cdots}}), \ldots, (t^{\overset{n-1}{\#\cdots}}, t^-) = (t^{\overset{n-1}{\#\cdots}}, t^{--}) + (t^{--}, t^-)$ is $\frac{n \cdot \beta \cdot N}{(n+1)(\beta+\gamma)}$. Therefore, $n/(n+1)$ of all commuters (N) will pay the toll to cross a queuing bottleneck under the optimal n-step toll schemes.*

(2) *The number of commuters who depart during the no toll period $[t_q, t^+)$ to avoid paying any tolls under the optimal n-step toll scheme in Table 9 is $\gamma \cdot N/((n+1)(\beta+\gamma))$. Because their departure times are earlier than $\hat{t}$, all of them will arrive at work early. On the other hand, there are two kinds of commuters who do not need to pay any tolls and arrive at work late under the optimal n-step toll schemes. One kind departs during the tolled period, but, as mentioned before, decides to stay at the roadside in front of the tollgate entry to the bottleneck from $t^{\overset{n}{\#\cdots}}$ until t^- to escape from being tolled. As shown in Table 10, the number of them during $[t^{\overset{n}{\#\cdots}}, t^-)$ is $\beta \cdot \gamma \cdot N/((n+1)(\alpha+\gamma)(\beta+\gamma))$. The other kind departs during the no-toll period $[t^-, t_{q'}]$ is $\alpha \cdot \beta \cdot N/((n+1)(\alpha+\gamma)(\beta+\gamma))$. Therefore, the total number of the two kinds of late arrivals is $\frac{\beta \cdot N}{(n+1)(\beta+\gamma)}$.*

Regular departure time switching decisions

Equilibrium departure time switching decisions from the no-toll to the optimal multi-step toll cases are shown as Table 11. These results are obtained according to 'the invariant schedule delay costs principle' that we have discussed in the optimal single-step toll case.

Commuters of Types I and IV in Table 11 represent the early and late arrivals, respectively, who will not alter their original departure times in the no-toll case if the optimal step-toll scheme is put into practice. Departure rate and departure interval for Type I are $\alpha \cdot s/(\alpha - \beta)$ and $[t_q, t']$, respectively, and for Type IV are $\alpha \cdot s/(\alpha + \gamma)$ and $[t^{\overset{n}{\#\cdots}}, t_{q'}]$, respectively, under the optimal n-step toll schemes (where $n = 1, 2, 3, \ldots$). On the other hand, commuters of Types II and III represent the early and later arrivals, respectively, who will alter their original departure times in the no-toll case if the optimal step toll scheme is put into practice. The switching routes to all toll payers ($\rho, 2\rho, 3\rho, \ldots, n\rho$) are illustrated in Table 11. In addition, the regularities of departure time switching decisions from the no-toll to the optimal n-step toll cases can be obtained by following the regular changes in departure time shifts under the optimal 1-, 2- and 3-step toll schemes.

Based on the outcomes shown in this Table 11, Proposition B, concerning investigation of how many groups of and what kind of commuters will make departure time switching decisions, is developed as follows.

PROPOSITION B: n groups of early arrivals and also n groups of late arrivals will alter their original departure times in the no-toll case if a queuing bottleneck is priced with the optimal n-step tolls. Moreover, commuters that will or will not alter their original departure times are

Table 4. *Departure time values under the optimal single- and multi-step toll schemes*

	1-step toll	2-step toll	3-step toll	n-step toll
t_q	0	0	0	0
$t_{q'}$	N/s	N/s	N/s	N/s
$\tilde{t}$	$\frac{\gamma(\alpha-\beta)}{\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{\gamma(\alpha-\beta)}{\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{\gamma(\alpha-\beta)}{\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{\gamma(\alpha-\beta)}{\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$
t^*	$\frac{\gamma}{\beta+\gamma}\left(\frac{N}{s}\right)$	$\frac{\gamma}{\beta+\gamma}\left(\frac{N}{s}\right)$	$\frac{\gamma}{\beta+\gamma}\left(\frac{N}{s}\right)$	$\frac{\gamma}{\beta+\gamma}\left(\frac{N}{s}\right)$
$\hat{t}$	$\frac{\gamma(2\alpha-\beta)}{2\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{\gamma(3\alpha-\beta)}{3\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{\gamma(4\alpha-\beta)}{4\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{\gamma[(n+1)\cdot\alpha-\beta]}{(n+1)\cdot\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$
t^+	$\frac{\gamma}{2\beta+\gamma}\left(\frac{N}{s}\right)$	$\frac{\gamma}{3\beta+\gamma}\left(\frac{N}{s}\right)$	$\frac{\gamma}{4(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{\gamma}{(n+1)(\beta+\gamma)}\left(\frac{N}{s}\right)$
t^{++}		$\frac{2\gamma}{3(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{2\gamma}{4(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{2\gamma}{(n+1)(\beta+\gamma)}\left(\frac{N}{s}\right)$
t^{+++}			$\frac{3\gamma}{4(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{3\gamma}{(n+1)(\beta+\gamma)}\left(\frac{N}{s}\right)$
$\vdots$				$\vdots$
$t^{\overset{n}{+\cdots}}$				$\frac{n\gamma}{(n+1)(\beta+\gamma)}\left(\frac{N}{s}\right)$
t^-	$\frac{\beta+2\gamma}{2(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{2\beta+3\gamma}{3(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{3\beta+4\gamma}{4(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{n\beta+(n+1)\cdot\gamma}{(n+1)(\beta+\gamma)}\left(\frac{N}{s}\right)$
t^{--}		$\frac{\beta+3\gamma}{3(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{2\beta+4\gamma}{4(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{(n-1)\cdot\beta+(n+1)\cdot\gamma}{(n+1)(\beta+\gamma)}\left(\frac{N}{s}\right)$
t^{---}			$\frac{\beta+4\gamma}{4(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{(n-2)\cdot\beta+(n+1)\cdot\gamma}{(n+1)(\beta+\gamma)}\left(\frac{N}{s}\right)$
$\vdots$				$\vdots$
$t^{\overset{n}{-\cdots}}$				$\frac{\beta+(n+1)\cdot\gamma}{(n+1)(\beta+\gamma)}\left(\frac{N}{s}\right)$
t'	$\frac{\gamma(\alpha-\beta)}{2\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{\gamma(\alpha-\beta)}{3\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{\gamma(\alpha-\beta)}{4\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{\gamma(\alpha-\beta)}{(n+1)\cdot\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$
t''		$\frac{\gamma(2\alpha-\beta)}{3\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{\gamma(2\alpha-\beta)}{4\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{\gamma(2\alpha-\beta)}{(n+1)\cdot\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$
t'''			$\frac{\gamma(3\alpha-\beta)}{4\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{\gamma(3\alpha-\beta)}{(n+1)\cdot\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$
$\vdots$				$\vdots$
$t^{\overset{n}{\prime\cdots}}$				$\frac{\gamma(n\alpha-\beta)}{(n+1)\cdot\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$
$t^{\#}$	$\frac{2\alpha\gamma+\alpha\beta-\beta\gamma}{2\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{3\alpha\gamma+\alpha\beta-\beta\gamma}{3\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{4\alpha\gamma+\alpha\beta-\beta\gamma}{4\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{(n+1)\cdot\alpha\gamma+\alpha\beta-\beta\gamma}{(n+1)\cdot\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$
$t^{\#\#}$		$\frac{3\alpha\gamma+2\alpha\beta-\beta\gamma}{3\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{4\alpha\gamma+2\alpha\beta-\beta\gamma}{4\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{(n+1)\cdot\alpha\gamma+2\alpha\beta-\beta\gamma}{(n+1)\cdot\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$
$t^{\#\#\#}$			$\frac{4\alpha\gamma+3\alpha\beta-\beta\gamma}{4\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{(n+1)\cdot\alpha\gamma+3\alpha\beta-\beta\gamma}{(n+1)\cdot\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$
$\vdots$				$\vdots$
$t^{\overset{n}{\#\cdots}}$				$\frac{(n+1)\cdot\alpha\gamma+n\alpha\beta-\beta\gamma}{(n+1)\cdot\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$
t_1		$\frac{2\gamma(\alpha-\beta)}{3\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{2\gamma(\alpha-\beta)}{4\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{2\gamma(\alpha-\beta)}{(n+1)\cdot\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$

(continued)

Table 4. *Continued*

	1-step toll	2-step toll	3-step toll	n-step toll
t_2			$\frac{3\gamma(\alpha-\beta)}{4\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$	$\frac{3\gamma(\alpha-\beta)}{(n+1)\cdot\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$
$\vdots$				$\vdots$
t_{n-1}				$\frac{n\gamma(\alpha-\beta)}{(n+1)\cdot\alpha(\beta+\gamma)}\left(\frac{N}{s}\right)$
$t_{1'}$		$\frac{1}{2}\left[\frac{2\beta(\alpha+\gamma)}{3\alpha(\beta+\gamma)}\right]\left(\frac{N}{s}\right)$	$\frac{1}{3}\left[\frac{3\beta(\alpha+\gamma)}{4\alpha(\beta+\gamma)}\right]\left(\frac{N}{s}\right)$	$\frac{1}{n}\left[\frac{n\beta(\alpha+\gamma)}{(n+1)\cdot\alpha(\beta+\gamma)}\right]\left(\frac{N}{s}\right)$
$t_{2'}$			$\frac{2}{3}\left[\frac{3\beta(\alpha+\gamma)}{4\alpha(\beta+\gamma)}\right]\left(\frac{N}{s}\right)$	$\frac{2}{n}\left[\frac{n\beta(\alpha+\gamma)}{(n+1)\cdot\alpha(\beta+\gamma)}\right]\left(\frac{N}{s}\right)$
$\vdots$				$\vdots$
$t_{(n-1)'}$				$\frac{n-1}{n}\left[\frac{n\beta(\alpha+\gamma)}{(n+1)\cdot\alpha(\beta+\gamma)}\right]\left(\frac{N}{s}\right)$

Table 5. *Equilibrium conditions under the optimal single- and multi-step toll schemes—cases for the early arrival at work*

	Equilibrium conditions	
	1-step toll scheme	2-step toll scheme
$t_q \le t < t'$	$\alpha T_Q(t)+\beta[t^*-(t+T_Q(t))]=\beta\cdot t^*$	$\alpha T_Q(t)+\beta[t^*-(t+T_Q(t))]=\beta\cdot t^*$
$t' \le t < t^+$	No one departs	No one departs
$t^+ \le t < \hat{t}$	$\alpha T_Q(t)+\beta[t^*-(t+T_Q(t))]+\rho=\beta\cdot t^*$	
$t^+ \le t < t''$		$\alpha T_Q(t)+\beta[t^*-(t+T_Q(t))]+\rho=\beta\cdot t^*$
$t'' \le t < t^{++}$		No one departs
$t^{++} \le t < \hat{t}$		$\alpha T_Q(t)+\beta[t^*-(t+T_Q(t))]+2\rho=\beta\cdot t^*$

	Equilibrium conditions	
	3-step toll scheme	n-step toll scheme
$t_q \le t < t'$	$\alpha T_Q(t)+\beta[t^*-(t+T_Q(t))]=\beta\cdot t^*$	$\alpha T_Q(t)+\beta[t^*-(t+T_Q(t))]=\beta\cdot t^*$
$t' \le t < t^+$	No one departs	No one departs
$t^+ \le t < t''$	$\alpha T_Q(t)+\beta[t^*-(t+T_Q(t))]+\rho=\beta\cdot t^*$	$\alpha T_Q(t)+\beta[t^*-(t+T_Q(t))]+\rho=\beta\cdot t^*$
$t'' \le t < t^{++}$	No one departs	No one departs
$t^{++} \le t < t'''$	$\alpha T_Q(t)+\beta[t^*-(t+T_Q(t))]+2\rho=\beta\cdot t^*$	$\alpha T_Q(t)+\beta[t^*-(t+T_Q(t))]+2\rho=\beta\cdot t^*$
$t''' \le t < t^{+++}$	No one departs	No one departs
$t^{+++} \le t < \hat{t}$	$\alpha T_Q(t)+\beta[t^*-(t+T_Q(t))]+3\rho=\beta\cdot t^*$	
$t^{+++} \le t < t''''$		$\alpha T_Q(t)+\beta[t^*-(t+T_Q(t))]+3\rho=\beta\cdot t^*$
$\vdots$		$\vdots$
$t'^{\overset{n}{\cdots}} \le t < t^{\overset{n}{+\cdots}}$		No one departs
$t^{\overset{n}{+\cdots}} \le t < \hat{t}$		$\alpha T_Q(t)+\beta[t^*-(t+T_Q(t))]+n\rho=\beta\cdot t^*$

those who will or will not pay the tolls, respectively, to cross a queuing bottleneck.

Proof:

(1) *According to the outcomes of Table 11, it is clear that there are n groups of Type II commuters change their original departure time periods to* $[t^+, t''), [t^{++}, t'''), \cdots, [t^{\overset{n}{+\cdots}}, \hat{t}\,)$ *under the optimal n-step toll scheme. Because their new departure time intervals are limited from* t^+ *until* $\hat{t}$, *they will pay the tolls and arrive at work early under the optimal n-step toll schemes. Besides that, also n groups of Type III commuters change their original departure time periods to* $(\hat{t}, t^{\overset{n}{-\cdots}}), [t^{\#}, t^{\overset{n-1}{-\cdots}}), \cdots, [t^{\overset{n-1}{\#\cdots}}, t^-)$. *Since their new departure time intervals are limited after* $\hat{t}$ *until* t^-, *they will pay the tolls and arrive at work late under the optimal n-step toll schemes. Therefore, all toll payers alter their original departure times in the no-toll case if*

Table 6. *Equilibrium conditions under the optimal single- and multi-step toll schemes—cases for the late arrival at work*

	Equilibrium conditions	
	1-step toll scheme	2-step toll scheme
$\hat{t} < t < t^{--}$		$\alpha T_Q(t) + \gamma[(t + T_Q(t)) - t^*] + 2\rho = \gamma(t_{q'} - t^*)$
$\hat{t} < t < t^{-}$	$\alpha T_Q(t) + \gamma[(t + T_Q(t)) - t^*] + \rho = \gamma(t_{q'} - t^*)$	
$t^{\#} \leq t < t^{--}$		$\alpha T_Q(t) + \gamma[(t + T_Q(t)) - t^*] + \rho = \gamma(t_{q'} - t^*)$
$t^{\#} \leq t < t^{-}$	$\alpha T_Q(t) + \gamma[(t + T_Q(t)) - t^*] = \gamma(t_{q'} - t^*)$	
$t^{--} \leq t < t^{-}$		$\alpha T_Q(t) + \gamma[(t + T_Q(t)) - t^*] + \rho = \gamma(t_{q'} - t^*)$
$t^{\#\#} \leq t < t^{-}$		$\alpha T_Q(t) + \gamma[(t + T_Q(t)) - t^*] = \gamma(t_{q'} - t^*)$
$t^{-} \leq t \leq t_{q'}$	$\alpha T_Q(t) + \gamma[(t + T_Q(t)) - t^*] = \gamma(t_{q'} - t^*)$	$\alpha T_Q(t) + \gamma[(t + T_Q(t)) - t^*] = \gamma(t_{q'} - t^*)$

	Equilibrium conditions	
	3-step toll scheme	*n*-step toll scheme
$\hat{t} < t < t^{\overset{n}{-\cdots}}$		$\alpha T_Q(t) + \gamma[(t + T_Q(t)) - t^*] + n\rho = \gamma(t_{q'} - t^*)$
$\hat{t} < t < t^{---}$	$\alpha T_Q(t) + \gamma[(t + T_Q(t)) - t^*] + 3\rho = \gamma(t_{q'} - t^*)$	
$t^{\#} \leq t < t^{\overset{n}{-\cdots}}$		$\alpha T_Q(t) + \gamma[(t + T_Q(t)) - t^*] + (n-1)\rho = \gamma(t_{q'} - t^*)$
$t^{\#} \leq t < t^{---}$	$\alpha T_Q(t) + \gamma[(t + T_Q(t)) - t^*] + 2\rho = \gamma(t_{q'} - t^*)$	
$t^{\overset{n}{-\cdots}} \leq t < t^{\overset{n-1}{-\cdots}}$		$\alpha T_Q(t) + \gamma[(t + T_Q(t)) - t^*] + (n-1)\rho = \gamma(t_{q'} - t^*)$
$t^{---} \leq t < t^{--}$	$\alpha T_Q(t) + \gamma[(t + T_Q(t)) - t^*] + 2\rho = \gamma(t_{q'} - t^*)$	
$t^{\#\#} \leq t < t^{\overset{n-1}{-\cdots}}$		$\alpha T_Q(t) + \gamma[(t + T_Q(t)) - t^*] + (n-2)\rho = \gamma(t_{q'} - t^*)$
$t^{\#\#} \leq t < t^{--}$	$\alpha T_Q(t) + \gamma[(t + T_Q(t)) - t^*] + \rho = \gamma(t_{q'} - t^*)$	
$t^{\overset{n-1}{-\cdots}} \leq t < t^{\overset{n-2}{-\cdots}}$		$\alpha T_Q(t) + \gamma[(t + T_Q(t)) - t^*] + (n-2)\rho = \gamma(t_{q'} - t^*)$
$t^{--} \leq t < t^{-}$	$\alpha T_Q(t) + \gamma[(t + T_Q(t)) - t^*] + \rho = \gamma(t_{q'} - t^*)$	
$t^{\#\#\#} \leq t < t^{-}$	$\alpha T_Q(t) + \gamma[(t + T_Q(t)) - t^*] = \gamma(t_{q'} - t^*)$	
$\vdots$		$\vdots$
$t^{\overset{n-1}{\#\cdots}} \leq t < t^{--}$		$\alpha T_Q(t) + \gamma[(t + T_Q(t)) - t^*] + \rho = \gamma(t_{q'} - t^*)$
$t^{--} \leq t < t^{-}$		$\alpha T_Q(t) + \gamma[(t + T_Q(t)) - t^*] + \rho = \gamma(t_{q'} - t^*)$
$t^{\overset{n}{\#\cdots}} \leq t < t^{-}$		$\alpha T_Q(t) + \gamma[(t + T_Q(t)) - t^*] = \gamma(t_{q'} - t^*)$
$t^{-} \leq t \leq t_{q'}$	$\alpha T_Q(t) + \gamma[(t + T_Q(t)) - t^*] = \gamma(t_{q'} - t^*)$	$\alpha T_Q(t) + \gamma[(t + T_Q(t)) - t^*] = \gamma(t_{q'} - t^*)$

a queuing bottleneck is priced with the optimal step toll schemes. The detailed departure time switching paths are shown in Table 11.

(2) *Also as shown in Table 11, commuters of Types I and IV are those who will not alter their original departure times in the no-toll case if a queuing bottleneck is priced with the optimal step toll scheme. Type I commuters depart during the no toll period* $[t_q, t')$, *so they do not need to pay any tolls. A part of Type IV commuters are those commuters who are not willing to enter the bottleneck from* $t^{\overset{n}{\#\cdots}}$ *until* t^{-} *to escape from being tolled that we have mentioned before. The other part of Type IV commuters are those who depart during the no-toll period* $[t^{-}, t_{q'}]$. *Therefore, all Type IV commuters do not need to pay any tolls.*

Finally, Proposition C, concerning to investigation of how long the toll payers (or departure time shifters) will postpone their original departure times, is developed as follows.

PROPOSITION C: A new departure time period to commuters who pay the first step toll ρ *under the optimal n-step toll scheme will be formed* $2\rho/(\alpha(n+1))$ *hours later than their original departure time period in the no-toll case. Moreover, departure time period postponements to other toll payers who pay* $2\rho, 3\rho, \cdots, n\rho$ *are* $2, 3, \cdots, n$ *times longer than the first step toll payer.*

Table 7. *The equilibrium commuting cost under the optimal single- and multi-step toll schemes—cases for the early arrival at work*

	1-step toll scheme			2-step toll scheme		
	EQC	ESDC	Toll	EQC	ESDC	Toll
$t_q \le t < t'$	$\frac{\alpha\beta t}{\alpha-\beta}$	$-\frac{\alpha\beta t}{\alpha-\beta}+\frac{\beta\gamma\cdot N}{(\beta+\gamma)s}$	0	$\frac{\alpha\beta t}{\alpha-\beta}$	$-\frac{\alpha\beta t}{\alpha-\beta}+\frac{\beta\gamma\cdot N}{(\beta+\gamma)s}$	0
$t' \le t < t^{+}$	0	0	0	0	0	0
$t^{+} \le t < \hat{t}$	$\frac{\alpha\beta t-\alpha\rho}{\alpha-\beta}$	$-\frac{\alpha\beta t}{\alpha-\beta}+\frac{\beta\gamma(2\alpha-\beta)N}{2(\beta+\gamma)(\alpha-\beta)s}$	ρ			
$t^{+} \le t < t''$				$\frac{\alpha\beta t-\alpha\rho}{\alpha-\beta}$	$-\frac{\alpha\beta t}{\alpha-\beta}+\frac{\beta\gamma(3\alpha-2\beta)N}{3(\beta+\gamma)(\alpha-\beta)s}$	ρ
$t'' \le t < t^{++}$				0	0	0
$t^{++} \le t < \hat{t}$				$\frac{\alpha\beta t-2\alpha\rho}{\alpha-\beta}$	$-\frac{\alpha\beta t}{\alpha-\beta}+\frac{\beta\gamma(3\alpha-\beta)N}{3(\beta+\gamma)(\alpha-\beta)s}$	2ρ

	3-step toll scheme			n-step toll scheme		
	EQC	ESDC	Toll	EQC	ESDC	Toll
$t_q \le t < t'$	$\frac{\alpha\beta t}{\alpha-\beta}$	$-\frac{\alpha\beta t}{\alpha-\beta}+\frac{\beta\gamma\cdot N}{(\beta+\gamma)s}$	0	$\frac{\alpha\beta t}{\alpha-\beta}$	$-\frac{\alpha\beta t}{\alpha-\beta}+\frac{\beta\gamma\cdot N}{(\beta+\gamma)s}$	0
$t' \le t < t^{+}$	0	0	0	0	0	0
$t^{+} \le t < t''$	$\frac{\alpha\beta t-\alpha\rho}{\alpha-\beta}$	$-\frac{\alpha\beta t}{\alpha-\beta}+\frac{\beta\gamma(4\alpha-3\beta)N}{4(\beta+\gamma)(\alpha-\beta)s}$	ρ	$\frac{\alpha\beta t-\alpha\rho}{\alpha-\beta}$	$-\frac{\alpha\beta t}{\alpha-\beta}+\frac{\beta\gamma[(n+1)\alpha-n\beta]N}{(n+1)(\beta+\gamma)(\alpha-\beta)s}$	ρ
$t'' \le t < t^{++}$	0	0	0	0	0	0
$t^{++} \le t < t'''$	$\frac{\alpha\beta t-2\alpha\rho}{\alpha-\beta}$	$-\frac{\alpha\beta t}{\alpha-\beta}+\frac{\beta\gamma(4\alpha-2\beta)N}{4(\beta+\gamma)(\alpha-\beta)s}$	2ρ	$\frac{\alpha\beta t-2\alpha\rho}{\alpha-\beta}$	$-\frac{\alpha\beta t}{\alpha-\beta}+\frac{\beta\gamma[(n+1)\alpha-(n-1)\beta]N}{(n+1)(\beta+\gamma)(\alpha-\beta)s}$	2ρ
$t''' \le t < t^{+++}$	0	0	0	0	0	0
$t^{+++} \le t < \hat{t}$	$\frac{\alpha\beta t-3\alpha\rho}{\alpha-\beta}$	$-\frac{\alpha\beta t}{\alpha-\beta}+\frac{\beta\gamma(4\alpha-\beta)N}{4(\beta+\gamma)(\alpha-\beta)s}$	3ρ			
$t^{+++} \le t < t''''$				$\frac{\alpha\beta t-3\alpha\rho}{\alpha-\beta}$	$-\frac{\alpha\beta t}{\alpha-\beta}+\frac{\beta\gamma[(n+1)\alpha-(n-2)\beta]N}{(n+1)(\beta+\gamma)(\alpha-\beta)s}$	3ρ
$\vdots$				$\vdots$	$\vdots$	$\vdots$
$\overset{n}{t^{\prime\cdots}} \le t < \overset{n}{t^{+\cdots}}$				0	0	0
$\overset{n}{t^{+\cdots}} \le t < \hat{t}$				$\frac{\alpha\beta t-n\alpha\rho}{\alpha-\beta}$	$-\frac{\alpha\beta t}{\alpha-\beta}+\frac{\beta\gamma[(n+1)\alpha-\beta]N}{(n+1)(\beta+\gamma)(\alpha-\beta)s}$	$n\rho$

Proof:

According to Tables 4 and 11, departure time period postponements to all toll payers in the optimal single- and multi-step toll cases when compared with their original departure time periods in the no-toll case can be obtained as follows:

(1) $t^{+}-t'=\hat{t}-\tilde{t}=(\rho/\alpha)$ *hours of departure time postponement to commuters who pay the unique toll,* ρ, *under the optimal 1-step toll scheme;*

(2) $t^{+}-t'=t^{\#}-t_{1'}=(2\rho/3\alpha)$ *hours of departure time postponement and* $t^{++}-t_1=\hat{t}-\tilde{t}=(4\rho/3\alpha)$ *hours of departure time postponement to commuters who pay* ρ *and* 2ρ, *respectively, under the optimal 2-step toll scheme;*

(3) $t^{+}-t'=t^{\#\#}-t_{2'}=(\rho/2\alpha)$ *hours of departure time postponement,* $t^{++}-t_1=t^{\#}-t_{1'}=(\rho/\alpha)$ *hours of departure time postponement and* $t^{+++}-t_2=\hat{t}-\tilde{t}=(3\rho/2\alpha)$ *hours of departure time postponement to commuters who pay* ρ, 2ρ *and* 3ρ, *respectively, under the optimal 3-step toll scheme;*

(4) $t^{+}-t'=\overset{n-1}{t^{\#\cdots}}-t_{(n-1)'}=(2\rho/(\alpha(n+1)))$ *hours of departure time postponement,* $t^{++}-t_1=\overset{n-2}{t^{\#\cdots}}-t_{(n-2)'}=(4\rho/(\alpha(n+1)))$ *hours of departure time postponement,* $t^{+++}-t_2=\overset{n-3}{t^{\#\cdots}}-t_{(n-3)'}=(6\rho/(\alpha(n+1)))_n$ *hour of departure time postponement,* $\cdots$, *and* $t^{+\cdots}-t_{n-1}=\hat{t}-\tilde{t}=(2n\rho/(\alpha(n+1)))$ *hours of departure time postponement to commuters who pay* ρ, 2ρ, $3\rho,\ldots,$ *and* $n\rho$, *respectively, under the optimal* n*-step toll schemes.*

Table 8. *The equilibrium commuting cost under the optimal single- and multi-step toll schemes—cases for the late arrival at work*

	1-step toll scheme			2-step toll scheme		
	EQC	ESDC	Toll	EQC	ESDC	Toll
$\hat{t} < t < t^{--}$				$\frac{\alpha\gamma(t_{q'} - t) - 2\alpha\rho}{\alpha + \gamma}$	$\frac{\alpha\gamma t}{\alpha + \gamma} - \frac{\gamma^2(3\alpha - \beta)N}{3(\beta + \gamma)(\alpha + \gamma)s}$	2ρ
$\hat{t} < t < t^{-}$	$\frac{\alpha\gamma(t_{q'} - t) - \alpha\rho}{\alpha + \gamma}$	$\frac{\alpha\gamma t}{\alpha + \gamma} - \frac{\gamma^2(2\alpha - \beta)N}{2(\beta + \gamma)(\alpha + \gamma)s}$	ρ			
$t^{\#} \le t < t^{---}$				$\frac{\alpha\gamma(t_{q'} - t) - \alpha\rho}{\alpha + \gamma}$	$\frac{\alpha\gamma t}{\alpha + \gamma} - \frac{\gamma^2(3\alpha - 2\beta)N}{3(\beta + \gamma)(\alpha + \gamma)s}$	ρ
$t^{\#} \le t < t^{-}$	$\frac{\alpha\gamma(t_{q'} - t)}{\alpha + \gamma}$	$\frac{\alpha\gamma t}{\alpha + \gamma} - \frac{\gamma^2(\alpha - \beta)N}{(\beta + \gamma)(\alpha + \gamma)s}$	0			
$t^{--} \le t < t^{-}$				$\frac{\alpha\gamma(t_{q'} - t) - \alpha\rho}{\alpha + \gamma}$	$\frac{\alpha\gamma t}{\alpha + \gamma} - \frac{\gamma^2(3\alpha - 2\beta)N}{3(\beta + \gamma)(\alpha + \gamma)s}$	ρ
$t^{\#\#} \le t < t^{-}$				$\frac{\alpha\gamma(t_{q'} - t)}{\alpha + \gamma}$	$\frac{\alpha\gamma t}{\alpha + \gamma} - \frac{\gamma^2(\alpha - \beta)N}{(\beta + \gamma)(\alpha + \gamma)s}$	0
$t^{-} \le t \le t_{q'}$	$\frac{\alpha\gamma(t_{q'} - t)}{\alpha + \gamma}$	$\frac{\alpha\gamma t}{\alpha + \gamma} - \frac{\gamma^2(\alpha - \beta)N}{(\beta + \gamma)(\alpha + \gamma)s}$	0	$\frac{\alpha\gamma(t_{q'} - t)}{\alpha + \gamma}$	$\frac{\alpha\gamma t}{\alpha + \gamma} - \frac{\gamma^2(\alpha - \beta)N}{(\beta + \gamma)(\alpha + \gamma)s}$	0

	3-step toll scheme			n-step toll scheme		
	EQC	ESDC	Toll	EQC	ESDC	Toll
$\hat{t} < y < t^{n}_{-\cdots}$				$\frac{\alpha\gamma(t_{q'} - t) - n\alpha\rho}{\alpha + \gamma}$	$\frac{\alpha\gamma t}{\alpha + \gamma} - \frac{\gamma^2[(n+1)\alpha - \beta]N}{(n+1)(\beta + \gamma)(\alpha + \gamma)s}$	$n\rho$
$\hat{t} < t < t^{---}$	$\frac{\alpha\gamma(t_{q'} - t) - 3\alpha\rho}{\alpha + \gamma}$	$\frac{\alpha\gamma t}{\alpha + \gamma} - \frac{\gamma^2(4\alpha - \beta)N}{4(\beta + \gamma)(\alpha + \gamma)s}$	3ρ			
$t^{\#} \le t < t^{n}_{-\cdots}$				$\frac{\alpha\gamma(t_{q'} - t) - (n-1)\alpha\rho}{\alpha + \gamma}$	$\frac{\alpha\gamma t}{\alpha + \gamma} - \frac{\gamma^2[(n+1)\alpha - 2\beta]N}{(n+1)(\beta + \gamma)(\alpha + \gamma)s}$	$(n-1)\rho$
$t^{\#} \le t < t^{---}$	$\frac{\alpha\gamma(t_{q'} - t) - 2\alpha\rho}{\alpha + \gamma}$	$\frac{\alpha\gamma t}{\alpha + \gamma} - \frac{\gamma^2(4\alpha - 2\beta)N}{4(\beta + \gamma)(\alpha + \gamma)s}$	2ρ			
$t^{n}_{-\cdots} \le t < t^{n-1}_{-\cdots}$				$\frac{\alpha\gamma(t_{q'} - t) - (n-1)\alpha\rho}{\alpha + \gamma}$	$\frac{\alpha\gamma t}{\alpha + \gamma} - \frac{\gamma^2[(n+1)\alpha - 2\beta]N}{(n+1)(\beta + \gamma)(\alpha + \gamma)s}$	$(n-1)\rho$
$t^{---} \le t < t^{--}$	$\frac{\alpha\gamma(t_{q'} - t) - 2\alpha\rho}{\alpha + \gamma}$	$\frac{\alpha\gamma t}{\alpha + \gamma} - \frac{\gamma^2(4\alpha - 2\beta)N}{4(\beta + \gamma)(\alpha + \gamma)s}$	2ρ			
$t^{\#\#} \le t < t^{n-1}_{-\cdots}$				$\frac{\alpha\gamma(t_{q'} - t) - (n-2)\alpha\rho}{\alpha + \gamma}$	$\frac{\alpha\gamma t}{\alpha + \gamma} - \frac{\gamma^2[(n+1)\alpha - 3\beta]N}{(n+1)(\beta + \gamma)(\alpha + \gamma)s}$	$(n-2)\rho$
$t^{\#\#} \le t < t^{--}$	$\frac{\alpha\gamma(t_{q'} - t) - \alpha\rho}{\alpha + \gamma}$	$\frac{\alpha\gamma t}{\alpha + \gamma} - \frac{\gamma^2(4\alpha - 3\beta)N}{4(\beta + \gamma)(\alpha + \gamma)s}$	ρ			
$t^{n-1}_{-\cdots} \le t < t^{n-2}_{-\cdots}$				$\frac{\alpha\gamma(t_{q'} - t) - (n-2)\alpha\rho}{\alpha + \gamma}$	$\frac{\alpha\gamma t}{\alpha + \gamma} - \frac{\gamma^2[(n+1)\alpha - 3\beta]N}{(n+1)(\beta + \gamma)(\alpha + \gamma)s}$	$(n-2)\rho$
$t^{--} \le t < t^{-}$	$\frac{\alpha\gamma(t_{q'} - t) - \alpha\rho}{\alpha + \gamma}$	$\frac{\alpha\gamma t}{\alpha + \gamma} - \frac{\gamma^2(4\alpha - 3\beta)N}{4(\beta + \gamma)(\alpha + \gamma)s}$	ρ			
$t^{\#\#\#} \le t < t^{-}$	$\frac{\alpha\gamma(t_{q'} - t)}{\alpha + \gamma}$	$\frac{\alpha\gamma t}{\alpha + \gamma} - \frac{\gamma^2(\alpha - \beta)N}{(\beta + \gamma)(\alpha + \gamma)s}$	0			
⋮				⋮	⋮	⋮
$t^{n-1}_{\#\cdots} \le t < t^{--}$				$\frac{\alpha\gamma(t_{q'} - t) - \alpha\rho}{\alpha + \gamma}$	$\frac{\alpha\gamma t}{\alpha + \gamma} - \frac{\gamma^2[(n+1)\alpha - n\beta]N}{(n+1)(\beta + \gamma)(\alpha + \gamma)s}$	ρ
$t^{--} \le t < t^{-}$				$\frac{\alpha\gamma(t_{q'} - t) - \alpha\rho}{\alpha + \gamma}$	$\frac{\alpha\gamma t}{\alpha + \gamma} - \frac{\gamma^2[(n+1)\alpha - n\beta]N}{(n+1)(\beta + \gamma)(\alpha + \gamma)s}$	ρ
$t^{n}_{\#\cdots} \le t < t^{-}$				$\frac{\alpha\gamma(t_{q'} - t)}{\alpha + \gamma}$	$\frac{\alpha\gamma t}{\alpha + \gamma} - \frac{\gamma^2(\alpha - \beta)N}{(\beta + \gamma)(\alpha + \gamma)s}$	0
$t^{-} \le t \le t_{q'}$	$\frac{\alpha\gamma(t_{q'} - t)}{\alpha + \gamma}$	$\frac{\alpha\gamma t}{\alpha + \gamma} - \frac{\gamma^2(\alpha - \beta)N}{(\beta + \gamma)(\alpha + \gamma)s}$	0	$\frac{\alpha\gamma(t_{q'} - t)}{\alpha + \gamma}$	$\frac{\alpha\gamma t}{\alpha + \gamma} - \frac{\gamma^2(\alpha - \beta)N}{(\beta + \gamma)(\alpha + \gamma)s}$	0

Table 9. *Equilibrium departure rates and the number of commuters under the optimal single- and multi-step toll schemes—cases for the early arrival at work*

	1-step toll scheme		2-step toll scheme	
	EDR	Number of commuters	EDR	Number of commuters
$t_q \le t < t'$	$\frac{\alpha \cdot s}{\alpha - \beta}$	$\frac{\gamma \cdot N}{2(\beta + \gamma)}$	$\frac{\alpha \cdot s}{\alpha - \beta}$	$\frac{\gamma \cdot N}{3(\beta + \gamma)}$
$t' \le t < t^{+}$	0	0	0	0
$t^{+} \le t < \hat{t}$	$\frac{\alpha \cdot s}{\alpha - \beta}$	$\frac{\gamma \cdot N}{2(\beta + \gamma)}$		
$t^{+} \le t < t''$			$\frac{\alpha \cdot s}{\alpha - \beta}$	$\frac{\gamma \cdot N}{3(\beta + \gamma)}$
$t'' \le t < t^{++}$			0	0
$t^{++} \le t < \hat{t}$			$\frac{\alpha \cdot s}{\alpha - \beta}$	$\frac{\gamma \cdot N}{3(\beta + \gamma)}$

	3-step toll scheme		n-step toll scheme	
	EDR	Number of commuters	EDR	Number of commuters
$t_q \le t < t'$	$\frac{\alpha \cdot s}{\alpha - \beta}$	$\frac{\gamma \cdot N}{4(\beta + \gamma)}$	$\frac{\alpha \cdot s}{\alpha - \beta}$	$\frac{\gamma \cdot N}{(n+1)(\beta + \gamma)}$
$t' \le t < t^{+}$	0	0	0	0
$t^{+} \le t < t''$	$\frac{\alpha \cdot s}{\alpha - \beta}$	$\frac{\gamma \cdot N}{4(\beta + \gamma)}$	$\frac{\alpha \cdot s}{\alpha - \beta}$	$\frac{\gamma \cdot N}{(n+1)(\beta + \gamma)}$
$t'' \le t < t^{++}$	0	0	0	0
$t^{++} \le t < t''$	$\frac{\alpha \cdot s}{\alpha - \beta}$	$\frac{\gamma \cdot N}{4(\beta + \gamma)}$	$\frac{\alpha \cdot s}{\alpha - \beta}$	$\frac{\gamma \cdot N}{(n+1)(\beta + \gamma)}$
$t''' \le t < t^{+++}$	0	0	0	0
$t^{+++} \le t < \hat{t}$	$\frac{\alpha \cdot s}{\alpha - \beta}$	$\frac{\gamma \cdot N}{4(\beta + \gamma)}$		
$t^{+++} \le t < t''''$			$\frac{\alpha \cdot s}{\alpha - \beta}$	$\frac{\gamma \cdot N}{(n+1)(\beta + \gamma)}$
$\vdots$			$\vdots$	$\vdots$
$t'^{n}_{\cdots} \le t < t^{n}_{+\cdots}$			0	0
$t^{n}_{+\cdots} \le t < \hat{t}$			$\frac{\alpha \cdot s}{\alpha - \beta}$	$\frac{\gamma \cdot N}{(n+1)(\beta + \gamma)}$

V. PRACTICAL IMPLICATIONS AND CONCLUSIONS

This paper has provided a methodological framework to forecast detailed commuter behaviour if a queuing road bottleneck is priced with the optimal single- and multi-step toll schemes. This kind of research is helpful to policy-makers because it is difficult to objectively forecast some uncertainties about commuters' decisions if the congestion toll is considered to put into practice. These uncertainties include how many and what kind of commuters will or will not pay the toll to cross a queuing bottleneck, and what are the changes in commuters' departure behaviour compared with the original no-toll case.

Some conclusions obtained from this paper and practical implications of these conclusions are illustrated as follows.

(1) This paper has shown the regularites in departure time values, equilibrium conditions, equilibrium commuting costs, equilibrium departure rates and the number of commuters under the optimal n-step toll schemes (where $n = 1, 2, 3, \ldots$). These regular results are useful references for decision-making and make the authorities easy to handle the optimal single- and multi-step toll schemes electronically.

(2) This paper has also shown all commuters' regular departure time switching decisions from the no-toll to the optimal n-step toll cases. According

Table 10. *Equilibrium departure rate and the number of commuters under the optimal single- and multi-step toll schemes—cases for the late arrival at work*

	1-step toll scheme		2-step toll scheme	
	EDR	Number of commuters	EDR	Number of commuters
$\hat{t} < t < t^{--}$			$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\beta \cdot N}{3(\beta + \gamma)}$
$\hat{t} < t < t^{-}$	$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\beta N}{2(\beta + \gamma)}$		
$t^{\#} \leq t < t^{--}$			$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\beta\gamma \cdot N}{3(a + \gamma)(\beta + \gamma)}$
$t^{\#} \leq t < t^{-}$	$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\beta\gamma \cdot N}{2(a + \gamma)(\beta + \gamma)}$		
$t^{--} \leq t < t^{-}$			$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\alpha\beta N}{3(a + \gamma)(\beta + \gamma)}$
$t^{\#\#} \leq t < t^{-}$			$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\beta\gamma \cdot N}{3(a + \gamma)(\beta + \gamma)}$
$t^{-} \leq t \leq t_{q'}$	$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\alpha\beta N}{2(a + \gamma)(\beta + \gamma)}$	$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\alpha\beta N}{3(a + \gamma)(\beta + \gamma)}$

	3-step toll scheme		n-step toll scheme	
	EDR	Number of commuters	EDR	Number of commuters
$\hat{t} < t < t_{-\cdots}^{n}$			$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\beta \cdot N}{(n+1)(\beta + \gamma)}$
$\hat{t} < t < t^{---}$	$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\beta\gamma \cdot N}{4(\beta + \gamma)}$		
$t^{\#} \leq t < t_{-\cdots}^{n}$			$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\beta\gamma \cdot N}{(n+1)(\alpha + \gamma)(\beta + \gamma)}$
$t^{\#} \leq t < t^{---}$	$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\beta\gamma \cdot N}{4(\alpha + \gamma)(\beta + \gamma)}$		
$t_{-\cdots}^{n} \leq t < t_{-\cdots}^{n-1}$			$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\alpha\beta N}{(n+1)(\alpha + \gamma)(\beta + \gamma)}$
$t^{---} \leq t < t^{--}$	$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\alpha\beta N}{4(\alpha + \gamma)(\beta + \gamma)}$		
$t^{\#\#} \leq t < t_{-\cdots}^{n-1}$			$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\beta\gamma \cdot N}{(n+1)(\alpha + \gamma)(\beta + \gamma)}$
$t^{\#\#} \leq t < t^{--}$	$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\beta\gamma \cdot N}{4(\alpha + \gamma)(\beta + \gamma)}$		
$t_{-\cdots}^{n-1} \leq t < t_{-\cdots}^{n-2}$			$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\alpha\beta N}{(n+1)(\alpha + \gamma)(\beta + \gamma)}$
$t^{--} \leq t < t^{-}$	$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\alpha\beta N}{4(\alpha + \gamma)(\beta + \gamma)}$		
$t^{\#\#\#} \leq t < t^{-}$	$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\beta\gamma \cdot N}{4(\alpha + \gamma)(\beta + \gamma)}$		
$\vdots$			$\vdots$	$\vdots$
$t_{\#\cdots}^{n-1} \leq t < t^{--}$			$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\beta\gamma \cdot N}{(n+1)(\alpha + \gamma)(\beta + \gamma)}$
$t^{--} \leq t < t^{-}$			$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\alpha\beta N}{(n+1)(\alpha + \gamma)(\beta + \gamma)}$
$t_{\#\cdots}^{n} \leq t < t^{-}$			$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\beta\gamma \cdot N}{(n+1)(\alpha + \gamma)(\beta + \gamma)}$
$t^{-} \leq t \leq t_{q'}$	$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\alpha\beta N}{4(\alpha + \gamma)(\beta + \gamma)}$	$\frac{\alpha \cdot s}{\alpha + \gamma}$	$\frac{\alpha\beta N}{(n+1)(\alpha + \gamma)(\beta + \gamma)}$

Table 11. *Departure time switching decisions from the no-toll case to the optimal single- and multi-step toll cases*

	No-toll→1-step toll	No-toll→2-step toll	No-toll→3-step toll	No-toll→n-step toll
Type I commuters $\left(DR = \frac{\alpha \cdot s}{\alpha - \beta}\right)$	Keep staying at $[t_{q'}, t')$	Keep staying at $[t_{q'}, t')$	Keep staying at $[t_q, t')$	Keep staying at $[t_q, t')$
Type II commuters $\left(DR = \frac{\alpha \cdot s}{\alpha - \beta}\right)$	$\rho : [t', \tilde{t}\,) \to [t^+, \hat{t}\,)$	$\rho \;: [t', t_1) \to [t^+, t'')$ $2p : [t_1, \tilde{t}) \to [t^{++}, \hat{t}\,)$	$\rho : [t', t_1) \to [t^+, t'')$ $2\rho \;: [t_1, t_2) \to [t^{++}, t'''')$ $3\rho : [t_2, \tilde{t}\,) \to [t^{+++}, \hat{t}\,)$	$\rho : [t', t_1) \to [t^+, t'')$ $2\rho : [t_1, t_2) \to [t^{++}, t''')$ $3\rho : [t_2, t_3) \to [t^{+++}, t'''')$ $\vdots$ $(n-1)\rho : [t_{n-2}, t_{n-1}) \to [t^{\overset{n-1}{+\cdots}}, t'^{\overset{n}{\cdots}})$ $n\rho : [t_{n-1}, \tilde{t}) \to [t^{\overset{n}{+\cdots}} \hat{t})$
Type III commuters $\left(DR = \frac{\alpha \cdot s}{\alpha + \gamma}\right)$	$\rho : (\tilde{t}, t^{\#}) \to (\hat{t}, t^-)$	$2\rho : (\tilde{t}, t_{1'}) \to (\hat{t}, t^-)$ $\rho : [t_{1'}, t^{\#\#}) \to [t^{\#}, t^-)$	$3\rho : (\tilde{t}, t_{1'}) \to (\hat{t}, t^{---})$ $2\rho : [t_{1'}, t_{2'}) \to [t^{\#}, t^-)$ $\rho : [t_{2'}, t^{\#\#\#}) \to [t^{\#\#}, t^-)$	$n\rho : (\tilde{t}, t_{1'}) \to (\hat{t}, t^{\overset{n}{-\cdots}})$ $(n-1)\rho : [t_{1'} t_{2'}) \to [t^{\#}, t^{\overset{n-1}{-\cdots}})$ $(n-2)\rho : [t_{2'} t_{3'}) \to [t^{\#\#}, t^{\overset{n-2}{-\cdots}})$ $\vdots$ $2\rho : [t_{(n-2)'}, t_{(n-1)'}) \to [t^{\overset{n-2}{\#\cdots}}, t^{--})$ $\rho : [t_{(n-1)'}, t^{\overset{n}{\#\cdots}}) \to [t^{\overset{n-1}{\#\cdots}}, t^-)$
Type IV commuters $\left(DR = \frac{\alpha \cdot s}{\alpha + \gamma}\right)$	Keep staying at $[t^{\#}, t_{q'}]$	Keep staying at $[t^{\#\#}, t_{q'}]$	Keep staying at $[t^{\#\#\#}, t_{q'}]$	Keep staying at $[t^{\overset{n}{\#\cdots}}, t_{q'}]$

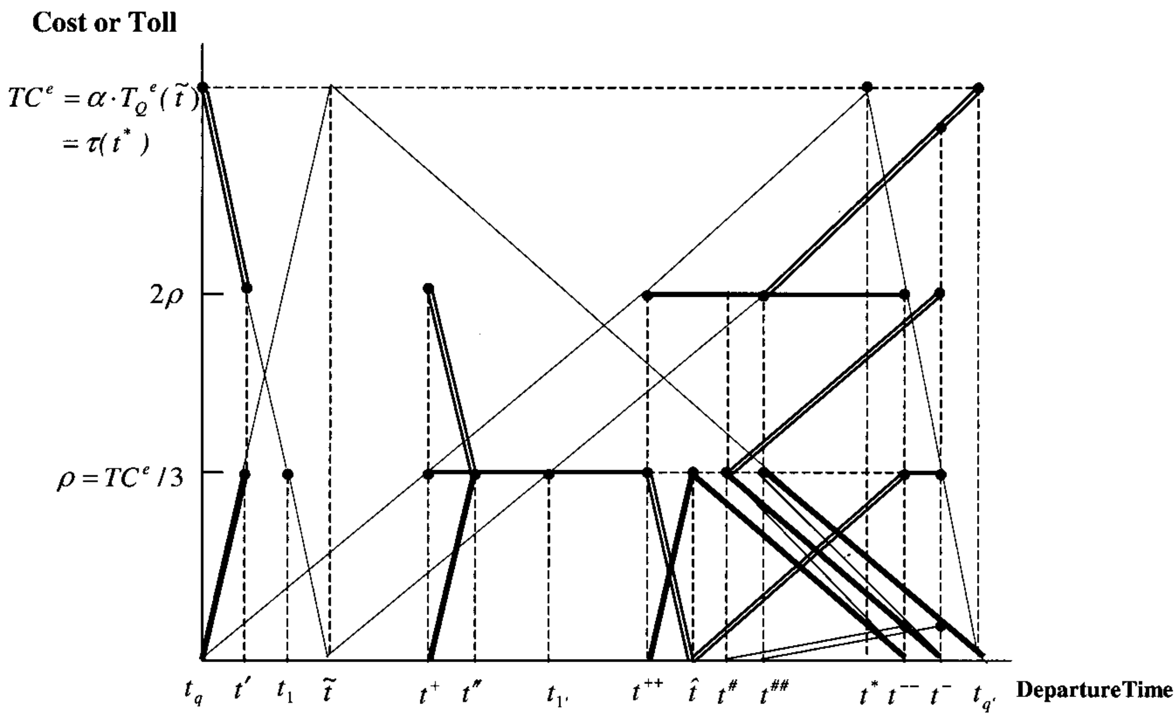

Fig. 6. *Equilibrium queuing costs and equilibrium schedule delay costs in the no-toll and optimal double-step toll cases*

to these results, we have found that $n/(n+1)$ of all commuters not only alter their original departure times in the previous no-toll case, but also pay the tolls to cross a queuing bottleneck. On the other hand, the remaining $1/(n+1)$ of all commuters neither alter their original departure time nor pay any tolls to cross a queuing bottleneck. The above information has two practical implications. First, it

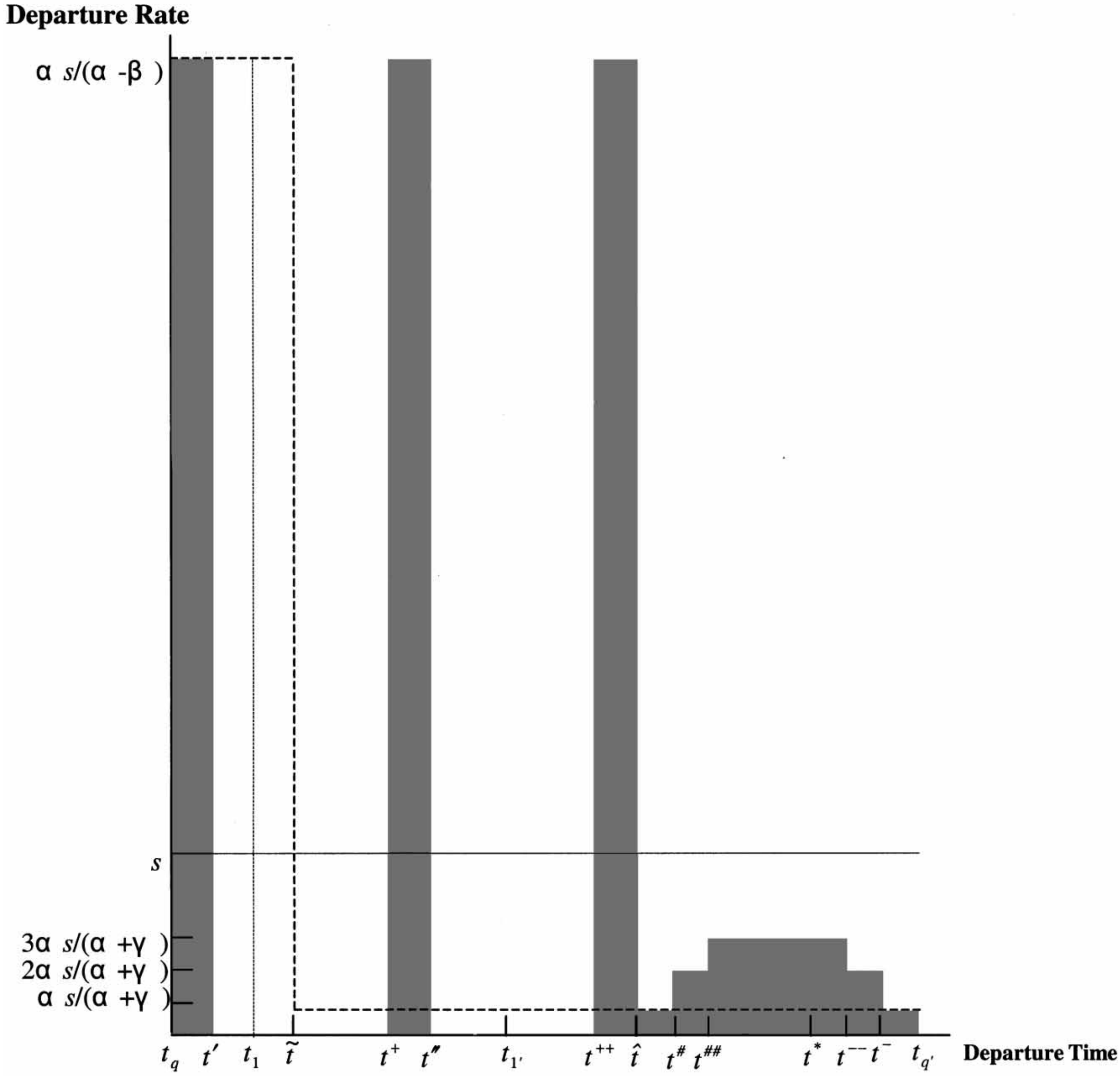

Fig. 7. *Equilibrium departure rates in the no-toll and optimal double-step toll cases*

allows policy-makers to estimate the toll revenue, which is paid by all departure time shifters, before implementing the optimal step toll scheme. Therefore, it is useful information for the authorities to budget the policy of levying the optimal step tolls at a queuing bottleneck. Second, it implies that the general characteristics of commuters (including percentages of different sexes, average ages, average waves, etc.) who will or will not pay the optimal step tolls become predictable in the present no-toll situation. Take the optimal 1-step toll case in Table 3, for example. The general characteristics of commuters who neither alter their original departure times nor pay any tolls to cross a queuing bottleneck can be investigated by surveying Types I and IV commuters who depart during the early arrival period $[t_q, t')$ and the late arrival period $[t^{\#}, t_{q'}]$, respectively, in the present no-toll situation. On the other hand, the general characteristics of commuters who are both the departure time shifter and the toll payer can be investigated by surveying Types II and III commuters who depart during the early arrival period $[t', \tilde{t})$ and the late arrival period $[\tilde{t}, t^{\#}]$, respectively, in the present no-toll situation.

(3) New departure times to the optimal n-step toll payers will be later than their original departure time in the no toll case. The length of time period between the original and new departure times to the first step toll (ρ) payer equals $2\rho/(\alpha(n+1))$. Moreover, the lengths to the second step toll (2ρ) payer, third step toll (3ρ) payer, $\cdots$, nth step toll ($n\rho$) are $2, 3, \ldots, n$ times longer than the first step toll payer. This implies two important things. First, the higher step tolls a commuter pays, the later he departs from home

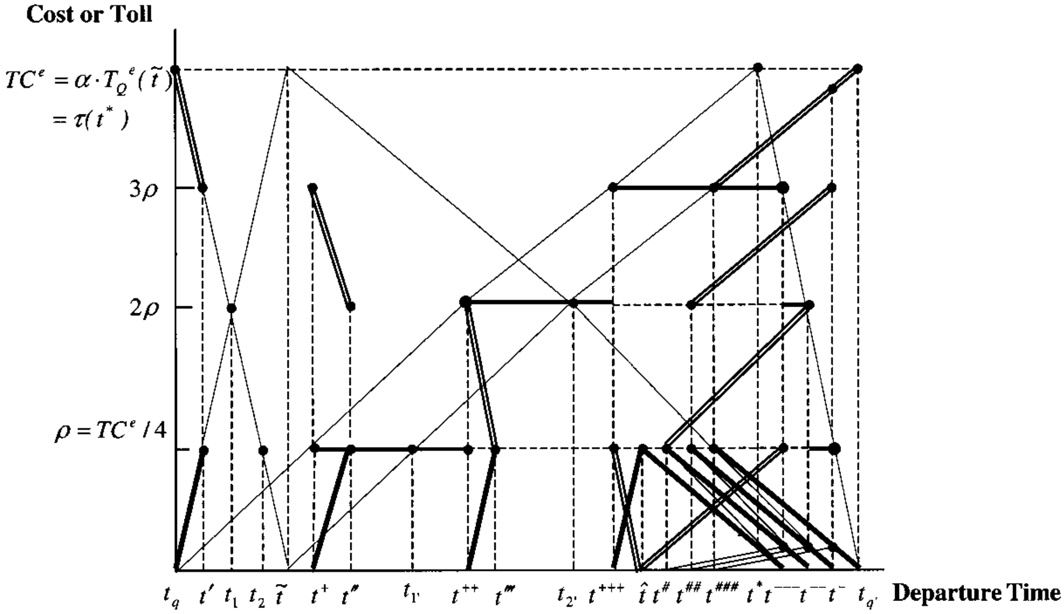

Fig. 8. *Equilibrium queuing costs and equilibrium schedule delay costs in the no-toll and optimal triple-step toll cases*

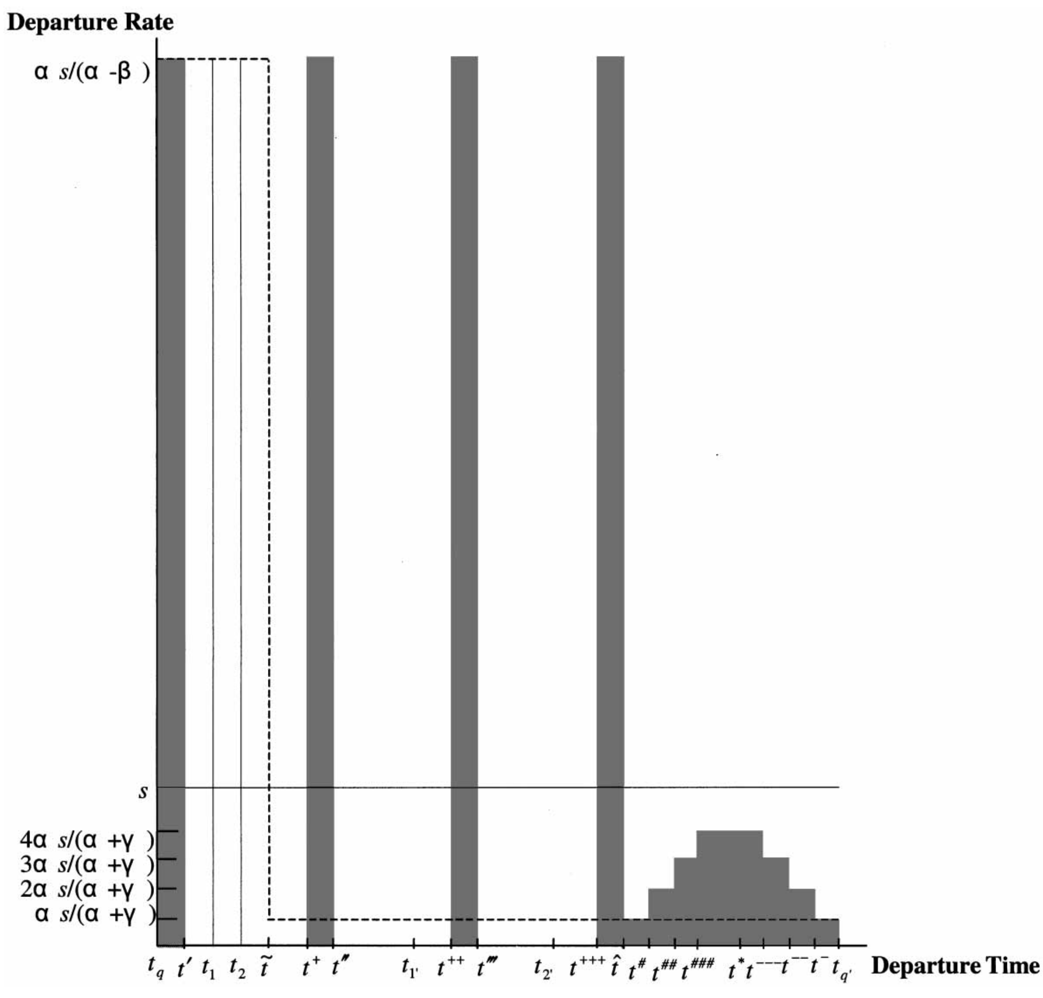

Fig. 9. *Equilibrium departure rates in the no-toll and optimal triple-step toll cases*

when compared with his original departure time in the no-toll case. Therefore, one can enjoy more leisure time at home before (s)he goes for work than in the original no-toll case if (s)he pays higher tolls under the optimal step toll scheme. Second, all possible lengths of the increased leisure time periods are computable as long as one decides to pay the optimal step toll to cross a queuing bottleneck.

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REFERENCES

Arnott, R. J., De Palma, A. and Lindsey, R. (1990) Economics of a bottleneck, *Journal of Urban Economics*, **27**, 111–30.

Arnott, R. J., De Palma, A. and Lindsey, R. (1993). A structural model of peak-load congestion: a traffic bottleneck with elastic demand, *American Economic Review*, **83**, 161–79.

Braid, M. (1989) Uniform versus peak-load pricing of a bottleneck with elastic demand, *Journal of Urban Economics*, **26**, 320–27.

Chen, O. and Bernstein, D. (1995) AM/PM congestion pricing with a single toll plaza, *Transportation Research Record*, No. 1498.

Cohen, Y. (1987) Commuter welfare under peak-load congestion tolls: who gains and who loses, *International Journal of Transport Economics*, **14**, 239–66.

De Palma, A. and Arnott, R. J. (1986). Usage-dependent peak-load pricing, *Economic Letter*, **20**, 101–5.

Laih, C.-H. (1994) Queuing at a bottleneck with single- and multi-step tolls, *Transportation Research*, **28A**, 197–208.

Laih, C.-H. (1998) Economic analysis of commuter behaviors under the road pricing, in *Proceedings of International Conference on Transportation into the Nex Millennium*. Centre for transportation Studies, Nanyang Technological University, Singapore, pp. 321–330.

Laih, C.-H. (2000) Equilibrium departure behavior under the optimal toll scheme in a queuing bottleneck, in *Proceedings of 9th International Association for Travel Behaviour Research Conference* (IATBR 2000), Gold Coast, Australia.

Newell, G. F. (1987) The morning commute problem for Nonidentical travelers, *Transportation Science*, **21**(2), 74–88.

Small, K. (1982). The scheduling of consumer activities, *American Economic Review*, **72**, 467–79.

Tabuchi, T. (1993) Bottleneck congestion and modal split, *Journal of Urban Economics*, **34**, 414–31.

Vickrey, W. S. (1969) Congestion theory and transport investment, *American economic review*, **59**, 251–61.

Yang, H. and Huang, H. J. (1997) Analysis of the time-varying pricing of a bottleneck with elastic demand using optimal control theory, *Transportation Research*, **31B**, 425–40.

Yang, H. and Meng, Q. (1998) Departure time, route choice and congestion toll in a queuing network with elastic demand, *Transportation Research*, **32B**, 247–60.

Vickrey, W.S. (1969) Congestion theory and transport investment, *American Economic Review*, **59**, 251–61.

APPENDIX

Definition of all notations in this paper.

t^* : a fixed work start time for all commuters;

t_q : time at which queue first forms;

$t_{q'}$: time at which queue disappears;

t : departure time from home, also arrival time at the bottleneck, where $t_q \le t \le t_{q'}$;

N : total number of auto-commuters who have to cross a bottleneck during $[t_q, t_{q'}]$ to reach their workplaces;

s : capacity of the bottleneck (measured by the traffic flow);

$\tilde{t}$: departure time at which allows one to arrive at work on time in the no-toll case;

$\hat{t}$: departure time at which allows one to arrive at work on time under the optimal step toll scheme;

τ : the optimal time-varying toll;

ρ : the optimal single-step toll, or the first step toll (the lowest toll) under the optimal multi-step toll schemes;

t^+ : time that starts the levying of the optimal single-step toll, or time that starts the levying of the first step toll under the optimal multi-step toll schemes;

t^{++} : time when the first step toll stops and the second step toll is levied under the optimal multi-step toll schemes;

t^{+++} : time when the second step toll stops and the third step toll is levied under the optimal multi-step toll schemes;

$t^{\overset{n}{+\cdots}}$: time when the $(n-1)$th step toll stops and the nth step toll is levied under the optimal n-step toll schemes;

t' : the start time when no one departs until t^+ under both the optimal single- and multi-step toll schemes;

t'' : the second start time when no one departs until t^{++} under the optimal multi-step toll schemes;

t''' : the third start time when no one departs until t^{+++} under the optimal multi-step toll schemes;

$t^{\overset{n}{\prime\cdots}}$: the nth start time when no one departs until $t^{\overset{n}{+\cdots}}$ under the optimal n-step toll schemes;

$t^{\overset{n}{-\cdots}}$: time when the nth step toll finishes and the $(n-1)$th step toll restarts under the optimal n-step toll schemes;

t^{---} : time when the third step toll finishes and the second step toll restarts under the optimal multi-step toll schemes;

t^{--} : time when the second step toll finishes and the first step toll restarts under the optimal multi-step toll schemes;

t^{-} : time when the optimal single-step toll finishes, or time when the first step toll finishes under the optimal multi-step toll schemes;

$t^{\#}$: time when some departures start waiting at the roadside until t^{-} to avoid paying optimal single-step toll, or time when some departures start waiting at the road side until $\overset{n}{t^{-\cdots}}$ in order to pay the $(n-1)$th step toll under the optimal multi-step toll schemes;

$t^{\#\#}$: time when some departures start waiting at the roadside until $\overset{n-1}{t^{-\cdots}}$ in order to pay the $(n-2)$th step toll under the optimal multi-step toll schemes;

$t^{\#\#\#}$: time when some departures start waiting at the roadside until $\overset{n-2}{t^{-\cdots}}$ in order to pay the $(n-3)$th step toll under the optimal multi-step toll schemes;

$\overset{n}{t^{\#\cdots}}$: time when some departures start waiting at the roadside until t^{-} in order to avoid paying any toll under the optimal multi-step toll schemes;

$t_1, t_2, t_3 \cdots t_n$: departure time before $\tilde{t}$ used to illustrate early arrivals' detailed departure time shifts from the not-toll to the optimal multi-step toll cases;

$t_{1'}, t_{2'}, t_{3'}, \cdots t_{n'}$: departure time after $\tilde{t}$ used to illustrate early arrivals' detailed departure time shifts from the not-toll to the optimal multi-step toll cases;

T_{Q} : time period spent waiting in the queue;

T_E : time period spent at the workplace before the work start time;

T_L : time period by which the work arrival time exceeds the work start time;

α : penalty cost per hour for the time period spent waiting in the queue;

β : penalty cost per hour for the time period spent at the workplace before the work start time;

γ : penalty cost per hour for the time period by which the arrival time at work exceeds the work start time;

TC : the commuting cost (per vehicle) incurred due to bottleneck queuing;

Superscript 'e' : equilibrium, e.g., TC^e, T_Q^e, T_E^e, T_L^e.