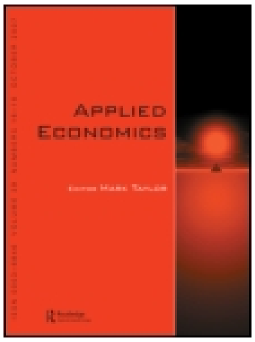

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When arriving at a busy port, a container ship usually need to queue in anchorage waiting for a berth. If the port establishes a toll scheme, the ship may rationally adjust sailing speed and time to save the cost, otherwise she has to pay for berthing on time. Consequently container ships' arrival times at the port will be dispersed, and the queuing situation can be decreased. In this article, an optimal step toll scheme by cost equilibrium approach is designed, and all values of equilibrium queuing cost and operating cost under the scheme are obtained. According to equilibrium results in this study, we show container ships that pay the toll to berth a queuing port are those altered their original arrival times at the port before the toll established. In addition, those ships' arrival time adjusting decisions from the nontoll to the optimal step toll cases can be well investigated before tolling a queuing port.

I. Introduction and Literature Review

When a container ship sails to the next appointed port, the shipmaster must notify the shipping company the time of arrival by telegraph or electronic mail. This procedure is convenient for the company to carry out dispatch steps in the container terminal. Cullinane and Song (2003) built a model concerning a productive and efficient container terminal. If arriving at a queuing port, a ship having to wait at anchorage until the berth available is frequent. This situation is very similar to commuting cars queuing in front of a road bottleneck in the morning rush hour. Each car has to queue for passing through the bottleneck one by one. Many transportation economic literatures have verified that this kind of queuing time can be efficiently reduced by collecting optimal queuing tolls. Hence, if the same theory applies to arrival container ships waiting at anchorage of the destination port, the satisfactory effect of optimal tolls on reducing the queuing can be expected.

A model of pricing a queuing at a bottleneck was initially developed by Vickrey (1969) and extended by Small (1982), De Palma and Arnott (1986), Braid (1989), Arnott *et al*. (1990, 1993), Tabuchi (1993), Laih (1994, 2000, 2001, 2004), Yang and Meng (1998). Among these researches, Laih (1994) first developed a flexible step pricing mechanism to relieve commuting queuing at a road bottleneck. Laih (2004) also provided a methodological framework to forecast the behaviour changes of commuters from a nontoll case to the optimal single-and multi-step toll cases. Applying these considerations, the optimal step toll scheme for a queuing port is derived in this article. With the scheme, arrival times of container ships will be rationally dispersed. Consequently, the queuing time for port entry can be decreased.

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This article also derives the consequent changes of container ships' arrival schedules after collecting the optimal step toll by economic equilibrium sense. Decisions of adjusting the arrival time from the nontoll to the tolled cases can be well investigated before a queuing port establishes the toll scheme. Port queuing pricing leads to the efficient use of berths especially during heavily congested periods. All of these are important issues for ship owners and port bureaus if the queuing pricing policy is considered by authorities.

Some literatures related to price a queuing bottleneck with the optimal time-varying toll and the optimal step toll schemes are reviewed as follows. Laih (1994) looked into the model of charging for queuing road bottleneck and used the static equilibrium analytical method to develop a series of optimal and sub-optimal step toll schemes. This development provided decision makers a set of collecting toll framework with flexibility when they tried to minimize negative effects of congestion at a bottleneck. Furthermore, Laih (2000) utilized the dynamic equilibrium analytical method to derive auto-commuters' moving tracks of departure time before and after the optimal single- and double-step toll schemes established. Next, Laih (2001) derived the optimal number of pricing steps, which minimized the total costs for the demand and supply sides, for the step toll scheme implemented at a queuing road bottleneck. These outcomes not only improved the theoretical model of the step toll scheme, but also provided authorities a rule operating the optimal step toll in practice. Moreover, Laih (2004) expanded the analysis of the optimal single- and double- steps to nth number ($n = 1, 2, 3, \ldots$) of charging steps. It was realized that when the charging steps increased one by one after detailed derivation, the framework of the optimal step toll, the related equilibrium costs, the equilibrium departure rates and moving tracks of departure time of auto-commuters had all shown regular variation. These complete and regular information not only facilitate policy makers to apply the optimal step toll scheme, but it can also be used to predict the entire auto-commuters' behaviour in the system of toll collection.

The framework of collecting the optimal step toll in this article is applicable to all general commercial ports. The nontoll equilibrium, the optimal time-varying toll, and the optimal step toll structure for a queuing port model are derived in Section II. Methodological frameworks used to forecast the changes of container ships' arrival time from the nontoll to the optimal step toll cases are developed in Section III. A numerical example explaining the frameworks mentioned in Section III is provided in Section IV. Finally, practical implications of the main results provided in this article are addressed in Section V.

II. The Model

Basic assumptions for a queuing port model are as follows. First, the assumed background in this article is that all berths are no vacancy and all container ships have to queue at anchorage until a vacant berth becomes available. Secondly, a large number of container ships will anchor at the port causing queuing during a certain period of time. This may be a result of increase in demand and supply of some goods attracting more ships calling the port, for example, export and import of Christmas merchandize at some ports from September to December. Thirdly, except this queuing port, there are no other alternative ports existed. Fourthly, apart from the queuing cost at the anchorage, other costs to a container ship in our model include the wharfage and the operating cost for containers stowed, discharged or loaded. Fifthly, the sequence of entering the port follows the principle of first in first serviced.

This article defines three possible schedules of arriving pattern at the destination port: on-time ($t + T_x(t) = \bar{t}$), early arrival ($t + T_x(t) < \bar{t}$) and late arrival ($t + T_x(t) > \bar{t}$) compared with the liner scheduled berthing time getting ready for cargo work in the port. Among these three situations, t is the time point when the container ship arrived at the anchorage of the port. $T_x(t)$ is the length of queuing time at the anchorage and varies in accordance with t. $\bar{t}$ is the scheduled berthing time at the port, called expected time of arrival (ETA) in this article.

According to the on-time schedule case in Fig. 1, $\tilde{t}$ is defined as the arrival time at the anchorage, which allows the ship berthing time is just on ETA after queuing. t^* is defined as the scheduled departure time at the port, called expected time of departure (ETD) in this article. The operating time length in unloading/loading cargoes required to meet ETD in this case is $t^* - \bar{t}$. Meanwhile, the wharfage to the ship is also counted from $\bar{t}$ to t^*. For the early schedule case in Fig. 1, $T_y(t)$ is defined as the time length of early arrival, i.e., $\bar{t} - (t + T_x(t))$, which implies that the ship berthing time is earlier than ETA. Because the early arrived container ship will not unload/load until $\bar{t}$ in order to avoid additional operating costs, the operating time needed to unload/load cargoes in this case is the same as the on-time schedule case. Meanwhile, the wharfage to the ship is

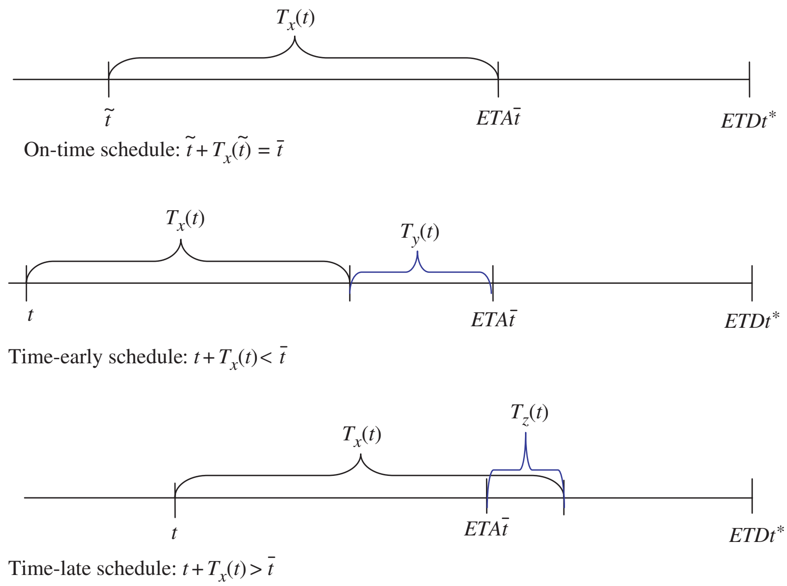

Fig. 1. Three possible schedules of arriving patterns at the destination port

counted from $(\bar{t}-T_y(t))$ to t^*. For the late scheduling case, on the other hand, $T_z(t)$ is defined as the time length of late arrival, i.e., $(t+T_x(t))-\bar{t}$, which implies that the ship berthing time is later than ETA. The length of both the operating time period and the wharfage period in this case is the same and equal to $t^*-(\bar{t}+T_z(t))$.

In order to develop the cost functions based on Fig. 1, we have the following assumptions. First, let x represent the waiting cost per hour to $T_x(t)$, then $xT_x(t)$ means the queuing time cost, which consists of personnel expense, depreciation cost of the ship, expense for repairing, insurance fee, interests, petrol fee for maintenance and desalination fee. These expenses are indispensable while ships anchor during the queuing period (t_1, t_2). t_1 and t_2 represent the start and the end times of queuing at the anchorage, respectively. t_1 will vary in accordance with the ship arrival times at the anchorage. With regard to t_2, it will vary in accordance with the time length required to disperse all queuing ships, and depend upon the number of ships queuing, the number of ships being able to be serviced and the entire operation time of a ship at the port in average. Secondly, let a represent the wharfage per hour to a container ship, b and b' represent the standard and additional operating time costs per hour, respectively. Especially to the container ship, it is reasonable to assume $b'>b$. When a container ship arrives late, the ship owner must bear the increased cost for unloading/loading cargoes, which includes increased costs to port hardware apparatus and longshoremen, in order to meet the scheduled ETD. Therefore, the additional operating time cost $b'T_z(t)$ is necessary to allow the ships that belong to the time-late scheduling case leave the port on ETD. Thirdly, generally speaking, $x>a$ and $(a+b)<b'$ are reasonable relationships in actual practice.

Accordingly, we obtain the following total cost $(C(t))$ to all container ships during the period from the time arriving and queuing at the anchorage until the time leaving the port:

(i) On-time schedule case

$$C(t)=x\cdot T_x(t)+(a+b)\cdot(t^*-\bar{t}) \quad \text{for } t=\tilde{t} \quad \text{or } t+T_x(t)=\bar{t} \tag{1}$$

(ii) Time-early schedule case:

$$\begin{aligned}C(t)&=x\cdot T_x(t)+a\cdot\left[T_y(t)+(t^*-\bar{t})\right]+b\cdot(t^*-\bar{t})\\&=x\cdot T_x(t)+a\cdot[t^*-(t+T_x(t))]+b\cdot(t^*-\bar{t})\\&\quad \text{for } t_1\le t+T_x(t)<\bar{t}\end{aligned} \tag{2}$$

(iii) Time-late schedule case:

$$\begin{aligned}C(t)&=x\cdot T_x(t)+(a+b)\cdot\left[t^*-(\bar{t}+T_z(t))\right]+b'\cdot T_z(t)\\&=x\cdot T_x(t)+(a+b)\cdot[t^*-(t+T_x(t))]\\&\quad +b'\cdot\left[(t+T_x(t))-\bar{t}\right] \quad \text{for } \bar{t}<t+T_x(t)\le t_2\end{aligned} \tag{3}$$

All ship owners have to face the same cost structure, as shown in (1)–(3), at a queuing port, the model is applicable to all queuing container ships.

Equilibrium obtains when no individual ship has an incentive to change the arrival time (t). This implies that the total cost $C(t)$ to each ship must be the same at all times during the queuing period (t_1, t_2). In other words, the equilibrium condition to the model is $\mathrm{d}C/\mathrm{d}t = 0$. For this purpose, we differentiate (1), (2) and (3) with t.

(i) On-time schedule case ($\tilde{t} + T_x(\tilde{t}) = \bar{t}$):
Unable to differentiate (1) because this case is a time spot ($\tilde{t}$).

(ii) Time-early schedule case ($t_1 \le t + T_x(t) < \bar{t}$):

$$\frac{\mathrm{d}C(t)}{\mathrm{d}t} = x\frac{\mathrm{d}T_x(t)}{\mathrm{d}t} + a\left(-1 - \frac{\mathrm{d}T_x(t)}{\mathrm{d}t}\right) = 0$$

$$\frac{\mathrm{d}T_x(t)}{\mathrm{d}t} = \frac{a}{x-a} \tag{4}$$

(iii) Time-late schedule case ($\bar{t} < t + T_x(t) \le t_2$):

$$\frac{\mathrm{d}C(t)}{\mathrm{d}t} = x \cdot \frac{\mathrm{d}T_x(t)}{\mathrm{d}t} - (a+b)\cdot\left(1 + \frac{\mathrm{d}T_x(t)}{\mathrm{d}t}\right) + b'\left(1 + \frac{\mathrm{d}T_x(t)}{\mathrm{d}t}\right) = 0$$

$$\frac{\mathrm{d}T_x(t)}{\mathrm{d}t} = \frac{a+b-b'}{x-a-b+b'} \tag{5}$$

(4) and (5) represent the slopes of the linear relationship between $T_x(t)$ and t. As mentioned before, $(x-a)$ in (4) is positive. On the other hand, $(x-a-b+b')$ and $(a+b-b')$ in (5) are positive and negative, respectively. Therefore, as shown in Fig. 2, the positive relationship occurs in early arrival situation and negative when ships arrive late. From this relationship, the equilibrium queuing time length can be easily calculated. Take $\bar{t}$ for example, $T_x(\bar{t})$ can be obtained as $(\bar{t} - t_2)\cdot(a+b-b')/(x-a-b+b')$.

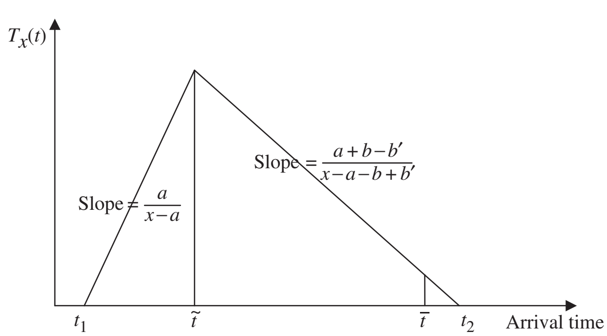

Fig. 2. The relationship between equilibrium queuing time length and arrival times

Suppose t_s is defined as the average time of stay at the berth for N arrival container ships during the queuing period (t_1, t_2), therefore we obtain

$$t_2 - t_1 = t_s \cdot (N-1) \tag{6}$$

According to $\tilde{t} + T_x(\tilde{t}) = \bar{t}$, the following Equations can be developed based on Fig. 2:

$$\tilde{t} + \frac{a}{x-a}\cdot(\tilde{t} - t_1) = \bar{t} \tag{7.1}$$

$$\tilde{t} + \frac{a+b-b'}{x-a-b+b'}\cdot(\tilde{t} - t_2) = \bar{t} \tag{7.2}$$

The decision we are now facing is how to locate $\tilde{t}$, t_1 and t_2 in equilibrium. By using (6) and (7), we obtained

$$\tilde{t} = \bar{t} + \frac{a(a+b-b')}{x(b'-b)}\cdot t_s \cdot (N-1) \tag{8.1}$$

$$t_1 = \bar{t} + \frac{a+b-b'}{b'-b}\cdot t_s \cdot (N-1) \tag{8.2}$$

$$t_2 = \bar{t} + \frac{a}{b'-b}\cdot t_s \cdot (N-1) \tag{8.3}$$

Since all container ships queuing during (t_1, t_2) have the same cost in equilibrium, by substituting $T_x(\tilde{t}) = \bar{t} - \tilde{t}$ and (8.1) into (1), or substituting $T_x(t_1) = 0$ and (8.2) into (2), or substituting $T_x(t_2) = 0$ and (8.3) into (3), the equilibrium total cost per ship can be expressed as

$$C^e = (a+b)(t^* - \bar{t}) + \frac{a(b'-a-b)}{b'-b}\cdot t_s \cdot (N-1) \tag{9}$$

Next, let us consider the toll scheme to a queuing port. The optimal time-varying toll is defined as a series of tolls that will completely eliminate the loss of queuing time without making ship owners worse off than they would be in the nontoll equilibrium. In order to attain such an objective, it is necessary to impose a series of tolls, $\Omega(t)$, that results in $T_x(t) = 0$ and $C(t) = C^e$ for all t in (1), (2) and (3). Then we obtain a series of the optimal time varying toll, $\Omega(t)$, listed in details below:

$$\Omega(t) = C^e - (a+b)(t^* - \bar{t}) = \frac{a\cdot(b'-a-b)}{b'-b}\cdot t_s\cdot(N-1) \quad \text{for} \quad t = \bar{t} \tag{10.1}$$

$$\Omega(t) = C^e - a\cdot(t^* - t) - b\cdot(t^* - \bar{t}) = \frac{a\cdot(b'-a-b)}{b'-b}\cdot t_s\cdot(N-1) - a\cdot(\bar{t} - t) \quad \text{for } t_1 \le t < \bar{t} \tag{10.2}$$

$$\Omega(t) = C^e - (a+b)(t^* - t) - b' \cdot (t - \bar{t}) = \frac{a \cdot (b' - a - b)}{b' - b} \cdot t_s \cdot (N-1) + (a + b - b') \cdot (t - \bar{t}) \quad \text{for } \bar{t} < t \leq t_2 \tag{10.3}$$

As shown in Fig. 3, the shape of the optimal time-varying toll scheme, $\Omega(t)$, is triangular t_1Bt_2 because of continuously changeable charges throughout the queuing period (t_1, t_2). The maximum optimal time-varying toll is located at $\bar{t}$ (= ETA). This is reasonable because ship owners are willing to pay the highest optimal time-varying toll to arrive on time without incurring any early or late arrival costs. The other triangular t_1At_2 in Fig. 3 represents the equilibrium queuing cost $xT_x(t)$. Because areas of two triangles are the same, the total optimal time varying toll completely replace all container ships' equilibrium queuing costs.

III. The Optimal Step Toll Scheme

The optimal time-varying toll is capable of eliminating queuing time completely, but has practical difficulties because it requires continuously changeable charges. Therefore a step toll scheme has been considered as an alternative to reduce queuing time. The step toll inscribed in the optimal fine toll triangle was first developed by Laih (1994) for reducing the queuing time to a desired level. As shown in Fig. 4, the amount of step toll is δ $(= \overline{GJ})$, and the revenue of the step toll scheme is shaped as t^+GJt^-. In general, a step toll structure with the maximum toll revenue inscribed in the optimal time-varying toll triangle is defined as the optimal step toll scheme to remove the largest proportion of the total queuing time. Take Fig. 4 for example to annotate this definition. If point G is moved to the left side horizontally and located on, say $\tilde{t}$, then the total cost to any container ship arriving during this extended tolled period $[\tilde{t}, t^+)$ will larger than her original nontoll equilibrium cost $C^e - (a+b)(t^* - \bar{t})$. The reason is that the ship is overtolled from $\Omega(t)$, where $t \in [\tilde{t}, t^+)$, up to δ. Hence only when $\overline{GJ}$ inscribed in the triangle t_1Bt_2 can obtain the maximum toll revenue and make ship owners no worse off than they would be in the nontoll equilibrium. Accordingly, the optimal step toll (δ) divides the maximum optimal time-varying toll $\Omega(\bar{t})$ into two equal amounts. Laih (1994) proved the existence of this structural characteristic. Laih (1994) also confirmed the effect of the optimal step toll on queuing reduction was 1/2 of the total queuing time that existed in the nontoll equilibrium. This is because the maximum toll revenues collected from the optimal step toll scheme is just half of the total equilibrium queuing costs in the nontoll case.

Equilibrium queuing costs

The optimal step toll, $\delta = \Omega(\bar{t})/2$, inscribed within the optimal time-varying toll t_1Bt_2 in Fig. 4, is applied at t^+ and lifted at t^-. On the other hand, the triangle t_1At_2 represents the nontoll equilibrium queuing time cost. Slopes of $\overline{t_1A}$, $\overline{At_2}$, $\overline{t_1R}$ and $\overline{Bt_2}$ can be easily obtained as $(a \cdot x)/(x-a)$, $x(a+b-b')/(x-a-b+b')$, a and $a+b-b'$, respectively from Figs 2 and 3 that have been proposed in Section II. $\hat{t}$ in Fig. 4 means the arrival time which allows the ship to berth on schedule (ETA) under the optimal step toll scheme.

In Fig. 4, t' and $t^{\#}$ are two important time spots. t' is defined as the start time when no ship arrives at the anchorage until t^+ under the optimal step toll scheme. $t^{\#}$ is defined as the time when some ships arrive at the

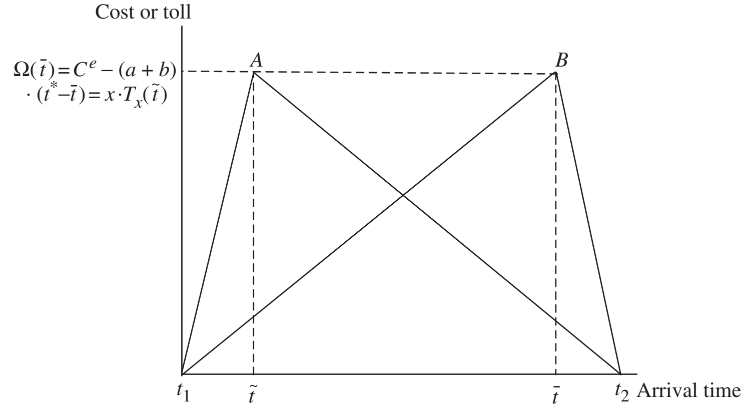

Fig. 3. The optimal time-varying toll t_1Bt_2 and equilibrium queuing cost t_1At_2

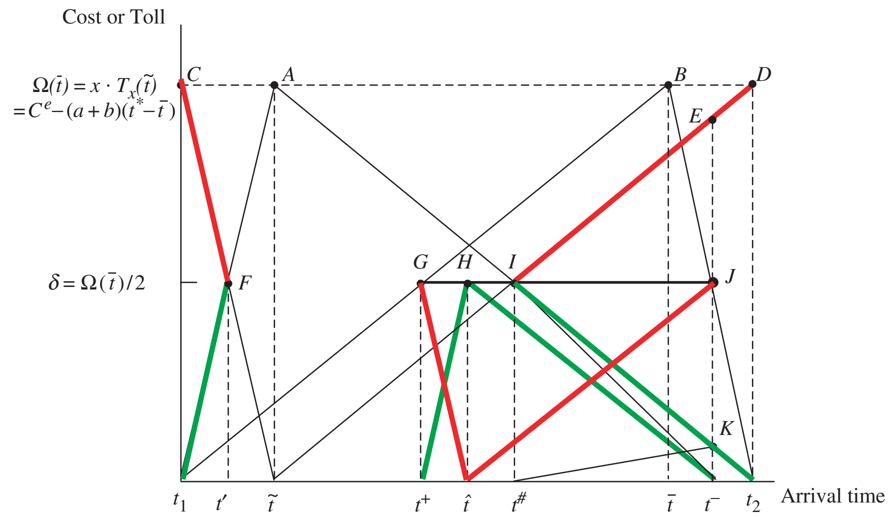

Fig. 4. Equilibrium queuing costs and equilibrium operating costs in the nontoll and optimal step toll cases

anchorage but decide not to notify the port officer for berthing until t^- in order to avoid paying the toll. Let's first interpret t'. Because the queuing cost to the first ship that will pay the toll δ at t^+ is zero, also because the arrival time at the berth to both the last untolled ship and the first tolled ship is almost the same, the last ship that will not pay the toll before t^+ must incur a queuing cost that is equal to the amount of the toll δ for the equilibrium purpose. Consequently the last untolled ship must arrive at the general anchorage δ/x earlier the first tolled ship. Hence there are no arrivals at the anchorage during the period $[t', t^+)$, and the length between t' and t^+ is equal to δ/x.

For the similar reason, because the queuing cost to the last ship that will pay the toll δ just before t^- is zero, also because the arrival time at the berth to both the last tolled ship and the first untolled ship is almost the same, the first ship that will not pay the toll after t^- must incur a queuing cost that is δ higher than the last tolled ship for the equilibrium purpose. This is impossible unless the first untolled ship has queued for a period of δ/x before t^-. Therefore, there are some arrival ships possibly waiting at the anchorage from $t^\#$ until t^- to avoid being tolled, and prepare to enter the berth free once the toll is lifted on t^-. Consequently, the length of the time period $[t^\#, t^-]$ is also equal to δ/x.

Note that t_1 is now assumed to locate on the origin (i.e. $t_1 = 0$) in Fig. 4 for the purpose of simplifying computation to other arrival time values without losing the generality. Detailed computations to values of the toll and arrival times appeared in Fig. 4 are shown in Table 1.

Table 2 illustrates equilibrium results for all arrival intervals under the optimal step toll scheme. Note that a blanket arrival time interval $[t', t^+)$ is existed because no ship arrives during this time period that mentioned earlier. Container ships of groups B, C and D arrive at the port during the tolled period $[t^+, t^-)$. Except group D escaping from being tolled, groups B and C pay the toll to enter the berth. Groups A and E do not need to pay the toll because they arrive during the no toll periods. In addition, only groups A and B will be alongside the berth earlier than ETA because they arrive at the anchorage before $\hat{t}$. These results are arranged as columns (I) ~ (III) in Table 2.

Since equilibrium will be achieved as long as all container ships have the same total cost throughout the queuing period, there are two kinds of equilibrium conditions for the early and late arrivals. One is $C(t) = C(t_1)$ for groups A and B of the early arrival, and the other is $C(t) = C(t_2)$ for groups C, D and E of the late arrival. These equilibrium conditions then can be expressed as follows:

$$x \cdot T_x(t) + a \cdot \left[t^* - (t + T_x(t))\right] + b \cdot \left(t^* - \bar{t}\right) = at^* + b\left(t^* - \bar{t}\right), \quad t_1 \le t \le t' \tag{11.1}$$

$$x \cdot T_x(t) + a \cdot \left[t^* - (t + T_x(t))\right] + b \cdot \left(t^* - \bar{t}\right) + \delta = at^* + b\left(t^* - \bar{t}\right), \quad t^+ \le t < \hat{t} \tag{11.2}$$

$$x \cdot T_x(t) + (a+b) \cdot [t^* - (t + T_x(t))] + b' \cdot \left[(t + T_x(t)) - \bar{t}\right] + \delta = (a+b)(t^* - t_2) + b'(t_2 - \bar{t}), \quad \hat{t} < t < t^- \tag{11.3}$$

Table 1. Values of the optimal step toll and arrival times appeared in Fig. 1

δ	$\delta = \frac{1}{2}(C^e - (a+b)(t^* - \bar{t})) = \frac{a(b' - a - b)}{2(b' - b)} \cdot t_s(N-1)$
t_1	$t_1 = 0$
t_2	$t_2 = t_s \cdot (N-1) - t_1 = t_s \cdot (N-1)$
$\tilde{t}$	$\tilde{t} = t_1 + \overline{t_1\tilde{t}} = 0 + \frac{C^e - (a+b)(t^* - \bar{t})}{ax/x - a} = \frac{(x-a)\cdot(b'-a-b)}{x\cdot(b'-b)} \cdot t_s(N-1)$
$\bar{t}$	$\bar{t} = \tilde{t} + T_x(\tilde{t}) = \tilde{t} + \frac{C^e - (a+b)(t^* - \bar{t})}{x} = \frac{b'-a-b}{b'-b} \cdot t_s(N-1)$
$\hat{t}$	$\hat{t} = \bar{t} - T_x(\hat{t}) = \bar{t} - \frac{\delta}{x} = \frac{(b'-a-b)(2x-a)}{2x(b'-b)} \cdot t_s(N-1)$
t^+	$t^+ = t_1 + \overline{t_1 t^+} = 0 + \frac{\delta}{a} = \frac{b'-a-b}{2(b'-b)} \cdot t_s(N-1)$
t'	$t' = t^+ - \frac{\delta}{x} = \frac{(x-a)(b'-a-b)}{2x\cdot(b'-b)} \cdot t_s(N-1)$
t^-	$t^- = t_2 - \overline{t^- t_2} = t_2 + \frac{\delta}{a+b-b'} = \frac{2b'-a-2b}{2(b'-b)} \cdot t_s(N-1)$
$t^\#$	$t^\# = t^- - \overline{t^\# t^-} = t^- - \frac{\delta}{x} = \frac{xa + (2x-a)(b'-a-b)}{2x(b'-b)} \cdot t_s(N-1)$

Table 2. Equilibrium results under the optimal step toll scheme

(I) Groups	(II) Arrival time intervals at the anchorage	(III) Types of arrivals at the berth	(IV) Equilibrium queuing costs (EQC): $x \cdot T_x(t)$ Equilibrium operating costs (EOC): $x \cdot T_x(\tilde{t}) - EQC - \delta$
A	$t_1 \le t < t'$	(a). Toll Free	$EQC = \frac{xat}{(x-a)}$
	(No toll period)	(b). Early Arrivals	$EOC = \frac{-xat}{(x-a)} + \frac{a(b'-a-b)}{(b'-b)} \cdot t_s(N-1)$
	$t' \le t < t^+$	None	$EQC = 0$
	(No toll period)		$EOC = 0$
B	$t^+ \le t < \hat{t}$	(a). Toll Payers	$EQC = \frac{xat}{(x-a)} - \frac{xa(b'-a-b)}{2(x-a)(b'-b)} \cdot t_s(N-1)$
	(Tolled period)	(b). Early Arrivals	$EOC = \frac{-xat}{(x-a)} + \frac{a(b'-a-b)(2x-a)}{2(x-a)(b'-b)} \cdot t_s(N-1)$
C	$\hat{t} < t < t^-$	(a). Toll Payers	$EQC = \frac{-x(b'-a-b)t}{(x-a-b+b')} + \frac{x(b'-a-b)(2b'-a-2b)}{2(x-a-b+b')(b'-b)} \cdot t_s(N-1)$
	(Tolled period)	(b). Late Arrivals	$EOC = \frac{x(b'-a-b)t}{(x-a-b+b')} - \frac{(b'-a-b)^2(2x-a)}{2(x-a-b+b')(b'-b)} \cdot t_s(N-1)$
D	$t^\# \le t < t^-$	(a). Toll Free	$EQC = \frac{-x(b'-a-b)t}{(x-a-b+b')} + \frac{x(b'-a-b)}{(x-a-b+b')} \cdot t_s(N-1)$
	(Tolled period)	(b). Late Arrivals	$EOC = \frac{x(b'-a-b)t}{(x-a-b+b')} - \frac{(b'-a-b)^2(x-a)}{(x-a-b+b')(b'-b)} \cdot t_s(N-1)$
E	$t^- \le t \le t_2$	(a). Toll Free	$EQC = \frac{-x(b'-a-b)t}{(x-a-b+b')} + \frac{x(b'-a-b)}{(x-a-b+b')} \cdot t_s(N-1)$
	(No toll period)	(b). Late Arrivals	$EOC = \frac{x(b'-a-b)t}{(x-a-b+b')} - \frac{(b'-a-b)^2(x-a)}{(x-a-b+b')(b'-b)} \cdot t_s(N-1)$

$$x \cdot T_x(t) + (a+b) \cdot [t^* - (t + T_x(t))] + b' \cdot \left[(t + T_x(t)) - \bar{t}\right] = (a+b)(t^* - t_2) + b'(t_2 - \bar{t}), \quad t^{\#} \le t < t^- \text{ or } t^- \le t \le t_2 \quad (11.4)$$

The above equilibrium conditions (11.1)–(11.4) are established to groups A, B, C and D (or E), respectively. The values of $C(t_1)$ and $C(t_2)$ are $at^* + b \cdot (t^* - \bar{t})$ and $(a+b)(t^* - t_2) + b' \cdot (t_2 - \bar{t})$, respectively, because $t_1 = 0$ and $T_x(t_1) = T_x(t_2) = 0$.

Equilibrium queuing costs (*EQC*: $x \cdot T_x(t)$), to groups A∼E, listed in column (IV) of Table 2 are obtained based on (11.1)–(11.4). As shown in Fig. 4, the equilibrium queuing costs to groups A ∼ E under the optimal step toll scheme are green lines $\overline{t_1F}$, $\overline{t^+H}$, $\overline{Ht^-}$, $\overline{IK}$ and $\overline{Kt_2}$, respectively. The slope of $\overline{t_1F}$ and $\overline{t^+H}$ for all early arrivals is $(ax/x - a)$, which is the same as the slope of the equilibrium queuing cost ($\overline{t_1A}$) to all early arrivals in the nontoll case. Note that there is no green lines of equilibrium queuing costs through the arrival period $[t', t^+)$ since no ship arrives during this period. In addition, the length of the queue will be reduced to zero at t^+ because of $T_x(t') = \delta/x = t^+ - t'$. On the other hand, the slope of $\overline{Ht^-}$, $\overline{IK}$ and $\overline{Kt_2}$ for all late arrivals is $(x \cdot (a+b-b'))/(x-a-b+b')$, which is the same as the slope of the equilibrium queuing cost ($\overline{At_2}$) to all late arrivals in the nontoll case. Note that group D ships' equilibrium queuing costs incurred before and after t^- in Fig. 4 are $\overline{It^-}$ and $\overline{t^{\#}K}$, respectively. The former It^- is the decreasing equilibrium queuing costs, from δ to zero, to those avoid paying the toll during the tolled period $[t^{\#}, t^-)$. The latter ($t^{\#}K$) is their increasing equilibrium queuing costs, from zero to $(a \cdot (b' - a - b) \cdot x \cdot t_s(N-1))/(2 \cdot (x - a - b + b') \cdot (b' - b))$ after t^-. Consequently, the total equilibrium queuing cost for group D ships is $\overline{IK} (= \overline{It^-} + \overline{t^{\#}K})$.

In Fig. 4, the total equilibrium queuing cost in the original nontoll case is $\Delta t_1 A t_2$, and the total equilibrium queuing cost under the optimal step toll scheme is composed of $\Delta t_1 F t'$, $\Delta t^+ H t^-$ and $\Delta t^{\#} I t_2$. Because $\Delta t^+ H t^-$ can be moved to ΔFAI due to the same area, it is then clear that the effect of the optimal step toll on queuing reduction is just half of the total queuing cost in the nontoll equilibrium.

Decisions of equilibrium arrival time adjustment

Because the optimal step toll derived from our model is simply the money cost to the toll payer who require to save the same amount of queuing costs, the equilibrium operating cost (*EOC*) in the optimal step toll case must be the same as that in the original nontoll case to maintain the equilibrium cost, $C^e - (a+b)(t^* - \bar{t}) = xT_x(\tilde{t}) = \Omega(\tilde{t})$. For this purpose, decisions of equilibrium arrival time adjustment of all container ships can be investigated by '*the invariant equilibrium operating cost principle*'.

In Table 2, the contents of the equilibrium total cost to groups A∼E under the optimal step toll scheme can be shown as

$$x \cdot T_x(t) + a \cdot \left[t^* - (t + T_x(t))\right] + b \cdot \left(t^* - \bar{t}\right),$$
$$x \cdot T_x(t) + a \cdot \left[t^* - (t + T_x(t))\right] + b \cdot \left(t^* - \bar{t}\right) + \delta,$$
$$x \cdot T_x(t) + (a+b) \cdot \left[t^* - (t + T_x(t))\right] + b' \cdot \left[(t + T_x(t)) - \bar{t}\right] + \delta,$$
$$x \cdot T_x(t) + (a+b) \cdot \left[t^* - (t + T_x(t))\right] + b' \cdot \left[(t + T_x(t)) - \bar{t}\right] \quad \text{and}$$
$$x \cdot T_x(t) + (a+b) \cdot \left[t^* - (t + T_x(t))\right] + b' \cdot \left[(t + T_x(t)) - \bar{t}\right],$$

respectively. Since the results of equilibrium queuing costs (*EQC*) to all arrival intervals under the optimal step toll scheme have been shown in column (IV) of Table 2, the corresponding values of *EOC* required to achieve the equilibrium cost, $C^e - (a+b)(t^* - \bar{t}) = xT_x(\tilde{t}) = \Omega(\tilde{t})$, can be easily obtained as shown in the same column. *EOC* for groups A∼E under the optimal step toll scheme are drawn as the red lines $\overline{CF}$, $\overline{G\hat{t}}$, $\overline{\hat{t}J}$, $\overline{IE}$ and $\overline{ED}$, respectively in Fig. 4. The slope of $\overline{CF}$ and $\overline{G\hat{t}}$ to all early arrivals is identical and equal to $(-a \cdot x/x - a)$. This is the same as the slope of $\overline{C\tilde{t}}$ that represents all early arrivals' *EOC* in the nontoll case. On the other hand, the slope of $\overline{\hat{t}J}$, $\overline{IE}$ and $\overline{ED}$ to all late arrivals is identical and equal to $(-x \cdot (a+b-b'))/(x-a-b+b')$. This is also the same as the slope of $\overline{\tilde{t}D}$ that represents all late arrivals' *EOC* in the nontoll case.

Next, the detailed discussion of all container ships' decisions to adjust arrival time will be made as follows. First, group A ships will not adjust their original arrival times in the nontoll case when the port is priced with the optimal step toll, because the equilibrium operating cost $\overline{CF}$ in both the nontoll and optimal step toll cases coincide during the arrival period (t_1, t'). Secondly, because the equilibrium operating cost ($\overline{G\hat{t}}$) in the optimal step toll case and the equilibrium operating cost ($\overline{F\tilde{t}}$) in the nontoll case are two identical and parallel lines, all group B ships that originally arrive during the period $(t', \tilde{t})$ in the nontoll case will adjust their arrivals to the period $(t^+, \hat{t})$ in the optimal step toll case. Similarly, because the equilibrium operating cost ($\overline{\hat{t}J}$) in the optimal step toll case and the equilibrium late cost ($\overline{\tilde{t}I}$) in the nontoll case are two identical and parallel lines, all group C ships that originally arrive during the period

Table 3. Ship owners' equilibrium decisions under the optimal step toll scheme

Types	Adjust arrival times or not	Toll payers or not	Early or late arrivals
Type I ships	No, keep arriving at $[t_1, t')$	No	Early
Type II ships	Yes, $[t', \tilde{t}) \rightarrow [t^+, \hat{t})$	Yes	Early
Type III ships	Yes, $[\tilde{t}, t^\#) \rightarrow [\hat{t}, t^-)$	Yes	Late
Type IV ships	No, keep arriving at $[t^\#, t_2]$	No	Late

$[\tilde{t}, t^\#]$ in the nontoll case will adjust their arrivals to the period $[\hat{t}, t^-]$ in the optimal step toll case. Thirdly, because the equilibrium operating cost $\overline{IE}$ in both the nontoll and optimal step toll cases coincide during the arrival time period $[t^\#, t^-]$, group D ships will not change their original arrival times in the nontoll case if the port is priced with the optimal step toll. Since $[t^\#, t^-]$ exists within $[\hat{t}, t^-]$, group C and D ships arrive simultaneously during $[t^\#, t^-]$. Finally, because the equilibrium operating cost $\overline{ED}$ in both the nontoll and optimal step toll cases coincide during the arrival time period $[t^-, t_2]$, group E ships will not change their original arrival times in the nontoll case if the port is priced with the optimal step toll.

The above outcomes are arranged and listed in Table 3. It is clear that ship owners who choose the same arrival times at the anchorage as they did in the original nontoll case are not the toll payers in the tolled case. These ships are Type I early arrivals as well as Type IV late arrivals in Table 3. The other part of container ships, i.e. Type II early arrivals and Type III late arrivals, that adjust their original arrival times at the anchorage are the toll payers.

IV. A Numerical Example

There is an assumptive case for explaining the above theory. According to the statistical data of Keelung port in Taiwan, The number of container ships in gross tonnage between 10 000 and 19 999 was 2520 accounting for 52% of the total container ships visiting the port in 2003. Therefore, the scenario of this case describes the ships of the above category arriving at Keelung port.

The wharfage (a) based on ship gross tonnage is charged to about US$66 per ship per hour for the category in accordance with the rate of Keelung port charge. The queuing time cost includes personnel expense of ship crew, fresh water fee, fuel oil fee, ship depreciation cost, insurance fee, etc. The fixed cost of a ship in gross tonnage between 10 000 and 19 999 is about US$3600 every day and listed in Table 4. Consequently the queuing time cost (x) is US$150 per hour.

Table 4. The distribution of ship fixed cost every day

Items	Cost ($USD)
Crew salaries (including salaries, meals, welfare, etc.)	500
Maintenance (including annual survey, repairing, maintaining, etc.)	320
Insurance (including hull insurance and P&I)	210
Spare parts	100
Fresh water	30
Lubricator	120
Ship depreciation	1400
Fuel oil consumption in port	620
Management	200
Other fixed cost	100
Total	3600

The average operation fee for loading/unloading a 20 feet container is about US$55 in Keelung port. The loading/unloading rate of one working gang is about 27 containers per hour. The standard operating time cost (b) is assumed to US$2970 per hour under two working gangs. The additional operating time cost (b') is assumed to 1.5 times of the standard operating time cost, about US$4455, in this case due to additional work for longshoremen and equipment.

According to the statistical records of Keekung port, the total berthing time of 5164 ships lying alongside piers were 50 215 hours in 2003. Therefore, the average berthing time 't_s' of each ship in the port is assumed to 10 hours.

The number of container ships (N) waiting in the anchorage is assumed to 5 ships. Then we have the following results: $t_s(N-1) = 10 \times (5-1) = 40$ hrs; $t_1 = 0{:}00$ (the first day); $t_2 = 16{:}00$ (the next day);

$$\rho = \frac{a(b' - a - b)}{2(b' - b)} \cdot t_s(N-1) = \text{US\$1261.33};$$

$$\tilde{t} = \frac{(x-a)(b'-a-b)}{x(b'-b)} \cdot t_s(N-1) = 21.40\,\text{hr}$$

(Arriving on time before the toll established);

$$\bar{t} = \frac{(b'-a-b)}{(b'-b)} \cdot t_s(N-1) = 38.22\,\text{hr} \quad \text{(ETA)};$$

$$\hat{t} = \frac{(b'-a-b)(2x-a)}{2x(b'-b)} \cdot t_s(N-1) = 29.81\,\text{hr}$$

(Arriving on time after the toll established);

$$t^{+} = \frac{(b'-a-b)}{2(b'-b)} \cdot t_s(N-1) = 19.11\,\text{hr};$$

$$t' = \frac{(x-a)\,(b'-a-b)}{2x(b'-b)} \cdot t_s(N-1) = 10.70\,\text{hr};$$

$$t^{-} = \frac{(2b'-a-2b)}{2(b'-b)} \cdot t_s(N-1) = 39.11\,\text{hr};$$

$$t^{\#} = \frac{xa + (2x-a)(b'-a-b)}{2x(b'-b)} \cdot t_s(N-1) = 30.70\,\text{hr}$$

The first ship arriving at the anchorage for waiting berth is the start of queuing, and the time is assumed at 00:00 of the first day. After 40 hours, that the last ship enters the port to berth means the end of queuing, the time is at 16:00 of the next day.

The optimal step toll is US$1261.33 per ship. The time of starting to pay the toll is at 19:07 of the first day, 19.11 hours after the queuing start time. The time of ending the toll is at 15:07, 39.11 hours after the queuing start time. The total time interval of paying the toll is 20 hours, just half of the queuing time. Based on the conclusion that we have obtained above, the effect of the optimal step toll on queuing reduction is just half of the total queuing time in the nontoll situation. In case the optimal step toll is established, the results of container ships changing arrival time at the anchorage and their actions are listed in Table 5. Notice that $[t', t^{+})$ is the time interval without any container ship arriving at the anchorage.

V. Practical Implications and Conclusions

In this article, an optimal step toll scheme by cost equilibrium approach is designed for a queuing port. All values of equilibrium queuing cost and operating cost and under the scheme are obtained. With this scheme, arrival times of container ships at the queuing port will be rationally dispersed. Consequently, the queuing time for port entry can be decreased. This article also derived the consequent changes of container ships' arrival schedules at the queuing port after collecting the optimal step toll. Decisions of adjusting the arrival time from the nontoll to the tolled cases can be well investigated before a queuing port establishes the toll scheme.

Some conclusions obtained from this article and their practical implications are illustrated as follows.

(1) A complete optimal step toll structure is established. All values of equilibrium queuing and operating costs under the optimal step toll scheme are derived. These results are useful for authorities to make decisions and to handle the scheme easily.

Table 5. Numerical results under the optimal step toll scheme

The time of container ships arriving at the anchorage before the toll established	The time of container ships arriving at the anchorage after the toll established	Actions of the ships after the toll established
$[t_1, t')$ 00:00 (the first day) ~ 10:42 (the first day)	$[t_1, t')$ no toll period 00:00 (the first day) ~ 10:42 (the first day)	(1) Keeping the same arrival time at the anchorage. (2) Arrival time at the berth is earlier than ETA. (3) No need to pay the toll.
$[t', \tilde{t})$ 10:42 (the first day) ~ 21:24 (the first day)	$[t^{+}, \hat{t})$ toll period 19:07 (the first day) ~ 05:49 (the next day)	(1) Adjusting the arrival time at the anchorage. (2) Arrival time at the berth is earlier than ETA. (3) Need to pay the toll.
$[\tilde{t}, t^{\#})$ 21:24 (the first day) ~ 06:42 (the next day)	$[\hat{t}, t^{-})$ toll period 05:49 (the next day) ~ 15:07 (the next day)	(1) Adjusting the arrival time at the anchorage. (2) Arrival time at the berth is later than ETA. (3) Need to pay the toll.
$[t^{\#}, t^{-})$ 06:42 (the next day) ~ 15:07 (the next day)	$[t^{\#}, t^{-})$ toll period 06:42 (the next day) ~ 15:07 (the next day)	(1) Keeping the same arrival time at the anchorage. (2) Arrival time at the berth is later than ETA. (3) No need to pay the toll (avoiding).
$[t^{-}, t_2]$ 15:07 (the next day) ~ 16:00 (the next day)	$[t^{-}, t_2]$ no toll period 15:07 (the next day) ~ 16:00 (the next day)	(1) Keeping the same arrival time at the anchorage. (2) Arrival time at the berth is later than ETA. (3) No need to pay the toll.

(2) Whether ship owners adjust ship arrival time will depend on the equilibrium operating cost. Ship owners who choose the same arrival times at the port as they did in the original nontoll case are not the toll payers in the tolled case. These ships are Type I early arrivals and Type IV late arrivals. The other parts of container ships, i.e. Type II early arrivals and Type III late arrivals that adjust their arrival times are the toll payers in the tolled case.
(3) According to the above results, the general characteristics to container ships whether pay the toll or not become predictable. For example, container ships neither adjust their original arrival times nor pay the toll at a queuing port can be investigated by surveying Type I and Type IV ships that arrive during the early arrival period $[t_2, t')$ and the late arrival period $[t^{\#}, t_2]$, respectively, before the toll established. On the other hand, ships of the other groups that are both the arrival time adjuster and the toll payer can be investigated by surveying Types II and III ships that arrive during the early arrival period $[t', \tilde{t})$ and the late arrival period $[\tilde{t}, t^{\#}]$, respectively.

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