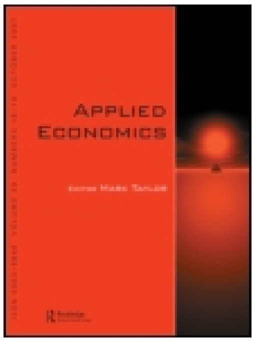

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Economics on the optimal port queuing pricing to bulk ships

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This article develops the optimal single step toll scheme which is levied to bulk ships for a queuing port. Bulk ships' arrival times at the port will be rationally dispersed after pricing this toll scheme. Consequently, the queuing time at the anchorage to all bulk ships will be rationally decreased. This article shows bulk ships' equilibrium arrival rate distributions during the queuing period before and after establishing the optimal single step toll scheme. This article also shows bulk ship owners' decisions of arrival time adjustments under the optimal single step toll scheme. Based on these results, we find some bulk ships that paid no toll under the optimal single step toll scheme to maintain the same arrival times at the anchorage as they did in the original nontoll equilibrium situation. New arrival times at the anchorage for other bulk ships that paid the toll are postponed when compared with their original arrival times in the nontoll equilibrium situation.

I. Introduction

When a bulk ship arrives at a queuing port, the port officers will guide her to wait at general anchorage. When a bulk ship berth becomes vacant and available, a pilot will shepherd her to the berth where it unloads and loads. Generally speaking, this situation is similar to auto-vehicles queuing at a road bottleneck especially in the morning rush hour. In order to make the efficient use of the bulk ship berth, this article develops a series of pricing schemes to bulk ships at a queuing port.

Laih (2008) has developed the optimal single- and multi-step toll schemes to container ships at a queuing port. Unfortunately, these toll schemes are not suitable for bulk ships. For this purpose, this article derives the optimal single step pricing to bulk ships at a queuing port. This article also derives the consequent changes of bulk ships' arrival schedules at the queuing port after collecting the optimal single step toll scheme. Consequently, bulk ship owners' arrival time adjusting decisions from the nontoll to the tolled cases can be predicted before the toll scheme is established at a queuing port. All of these are important issues for bulk ship owners and the port bureau if the port queuing pricing to bulk ships is considered to put into practice by the authorities.

The related literatures concerning to pricing a queuing bottleneck are raised as follows. A model of pricing a queuing bottleneck was first developed by Vickrey (1969). In his model, nontoll equilibrium is illustrated and the optimal variable toll, which eliminates queuing completely, is determined to stagger commuters' departure times. Small (1982) derived and estimated a disaggregate econometric model for the choice of trip schedule by automobile commuters. He found that, on average, urban commuters will shift their schedules by 1 o 2 minutes

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towards the early side, or by 1/3 to 1 minutes towards the late side, in order to save a minute of travel time. Laih (1994) developed a flexible pricing mechanism including the optimal single- and multi-step tolls to relieve commuting queuing in the morning at a road bottleneck. He showed that $n/(n+1)$ of the total queuing time that exists under the nontoll equilibrium can be eliminated with the optimal *n-step* toll. Laih (2004) provided a methodological framework to forecast commuter behaviour changes from the no toll case to both the optimal single- and multi-step toll cases. When the charging steps increased one by one, the framework of the single optimal step toll, the related equilibrium costs, the equilibrium departure rates and moving tracks of departure time of auto-commuters vary regularly. These complete and regular information not only facilitate policy makers for the application of the optimal single step toll scheme, but it can also be used to predict the entire auto-commuters' behaviour in the system of toll collection. Moreover, Laih *et al.* (2006) and Laih and Chen (2007) designed an optimal single- and multi-step toll schemes, respectively, to container ships by cost equilibrium approach for a queuing port. All values of equilibrium queuing cost, operating cost and in patterns of equilibrium arrival time shifted under the optimal single- and multi-step toll schemes are obtained. With these toll schemes, arrival times of container ships at the queuing port will be rationally dispersed. Consequently, the queuing time for port entry can be decreased. These papers also derived the consequent changes of container ships' arrival schedules at the queuing port after collecting the optimal single- and multi-step tolls. Decisions of adjusting the arrival time from the nontoll to the tolled cases can be predicted before these toll schemes are established. Unfortunately, however, the above results are not suitable for bulk ships at a queuing port. This problem will be dealt within the following sections.

The rest of the article is structured as follows. The basic model including the equilibrium total cost and the optimal time-varying toll scheme to bulk ships at a queuing port are derived in Section II. Based on the basic model, Section III develops the optimal single step toll scheme, inscribed within the optimal time-varying toll scheme, to bulk ships for practical purposes. Equilibrium queuing costs to bulk ships and bulk ships' equilibrium arrival rates at the anchorage under the optimal single step toll scheme are derived in Section IV. A framework used to predict bulk ship owners' decisions of arrival time adjustments from the nontoll to the tolled cases are established in Section V. Finally, the main results provided in this article are addressed in Section VI.

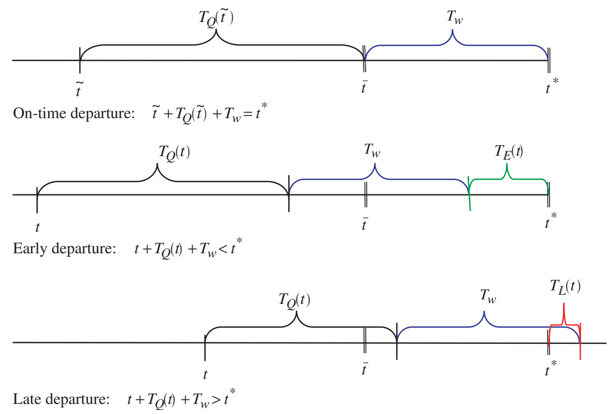

Fig. 1. Bulk ships' departure patterns at a queuing port

II. The Basic Model

Basic assumptions for a queuing port model are as follows. First, the assumed background in this article is that arrived bulk ships have to queue at the anchorage until a vacant berth for cargo loading/unloading becomes available. This may be a result of increase in demand or supply of bulk cargos attracting more ships calling the port. Secondly, the sequence of entering the berth at a queuing port follows the principle of first come first serviced. Thirdly, except this queuing port, there are no other alternative ports existed. Fourth, apart from the queuing cost at the anchorage, other costs to a bulk ship owner in our model include the wharfage for a bulk ship alongside the berth, the dispatch fee or revenue loss due to early or late departure from the queuing port, respectively, and the toll (if any).

In Fig. 1, There are three kinds of departure patterns to all bulk ships: on-time departure $(\tilde{t} + T_Q(\tilde{t}) + T_w = t^*)$, early departure $(t + T_Q(t) + T_w < t^*)$ and late departure $(t + T_Q(t) + T_w > t^*)$. Among these three situations, $\bar{t}$ and t^* are Estimate Time of Arrival (ETA) and Estimate Time of Departure (ETD), respectively, to all bulk ships. $\tilde{t}$ is defined as a bulk ship's arrival time at the anchorage which allows the berthing time just the same as ETA after queuing. t is the time point when a bulk ship arrives at the anchorage of the port. $T_Q(t)$ is the length of queuing time period at the anchorage and varies in accordance with t. For simplicity, $T_Q(t)$ is assumed to be a linear function. $T_w = t^* - \bar{t}$ is the average (fixed) operation time length that each bulk ship staying at a berth for all cases in Fig. 1. $T_E(t) = t^* - (t + T_Q(t) + T_w)$ and $T_L(t) = (t + T_Q(t) + T_w) - t^*$ in Fig. 1 are defined as the time lengths of early departure period and late departure period, respectively.

According to Fig. 1, we obtain the total costs to all bulk ships from the time arriving and queuing at the anchorage until the time leaving the port:
on-time departure:

$$\begin{aligned} TC(\tilde{t}) &= \alpha \cdot T_Q(\tilde{t}) + w \cdot T_w \\ &= \alpha \cdot T_Q(\tilde{t}) + w \cdot (t^* - \tilde{t}) \\ \tilde{t} &+ T_Q(\tilde{t}) + T_w = t^* \end{aligned} \tag{1}$$

early departure:

$$\begin{aligned} TC(t) &= \alpha \cdot T_Q(t) + w \cdot T_w + \beta \cdot T_E(t) \\ &= \alpha \cdot T_Q(t) + w \cdot T_w \\ &\quad + \beta \cdot [t^* - (t + T_Q(t) + T_w)] \\ t_q &\le t + T_Q(t) + T_w < t^* \end{aligned} \tag{2}$$

late departure:

$$\begin{aligned} TC(t) &= \alpha \cdot T_Q(t) + w \cdot T_w + \gamma \cdot T_L(t) \\ &= \alpha \cdot T_Q(t) + w \cdot T_w \\ &\quad + \gamma \cdot [(t + T_Q(t) + T_w) - t^*] \\ t^* &< t + T_Q(t) + T_w \le t_{q'} \end{aligned} \tag{3}$$

In (1)–(3), α represents the time cost per hour to $T_Q(t)$, then $\alpha T_Q(t)$ means the queuing time cost, which consists of personnel expense, depreciation cost of the ship, expense for repairing, insurance fee, interests, petrol fee for maintenance and desalination fee. These expenses are indispensable while ships queue at the anchorage during the queuing period $[t_q, t_{q'}]$, where t_q and $t_{q'}$ represent the start and the end times of queuing at the anchorage, respectively. w represents the wharfage per hour for T_w to a bulk ship, the total wharfage $w \cdot T_w$ should be paid by bulk ship owners in most cases of tramp shipping. In (2) and (3), β and γ represent the time cost per hour for T_E and T_L, respectively. In tramp shipping, the ship owner should pay the dispatch fee to the consignor if the ship's departure time becomes earlier than ETD due to the consignor's prompt arrangements for cargos being loaded/unloaded. For the early departure case in Fig. 1, the dispatch fee is $\beta \cdot [t^* - (t + T_Q(t) + T_w)]$. On the other hand, the ship owner has to suffer revenue losses on delaying the next voyage if the ship's departure time becomes later than ETD. For the late departure case in Fig. 1, the revenue loss is $\gamma \cdot [(t + T_Q(t) + T_w) - t^*]$.

Equilibrium obtains when no individual bulk ship has an incentive to change the arrival time, t. This implies that the total cost $TC(t)$ to each bulk ship must be the same at all times during the queuing period $[t_q, t_{q'}]$. In other words, the equilibrium condition is $\mathrm{d}TC/\mathrm{d}t = 0$. For this purpose, differentiating (2) and (3) with respect to t, then we obtain (4) and (5), respectively, as follows:

$$\frac{\mathrm{d}TC(t)}{\mathrm{d}t} = \alpha \cdot \frac{\mathrm{d}T_Q(t)}{\mathrm{d}t} + \beta \cdot \left[-1 - \frac{\mathrm{d}T_Q(t)}{\mathrm{d}t}\right] = 0$$

$$\frac{\mathrm{d}T_Q(t)}{\mathrm{d}t} = \frac{\beta}{\alpha - \beta} \quad \text{for } t_q \le t + T_Q(t) + T_w < t^* \tag{4}$$

$$\frac{\mathrm{d}TC(t)}{\mathrm{d}t} = \alpha \cdot \frac{\mathrm{d}T_Q(t)}{\mathrm{d}t} + \gamma \cdot \left[1 + \frac{\mathrm{d}T_Q(t)}{\mathrm{d}t}\right] = 0$$

$$\frac{\mathrm{d}T_Q(t)}{\mathrm{d}t} = \frac{-\gamma}{\alpha + \gamma} \quad \text{for } t^* < t + T_Q(t) + T_w \le t_{q'} \tag{5}$$

Because $(\alpha - \beta)$ in (4) is positive in practice, (4) and (5) represent the positive and negative slopes of the linear relationship between $T_Q(t)$ and t in early and late departure situations, respectively. Using (4) and (5), the equilibrium queuing time length can be easily calculated. Take $\bar{t}$ for example, $T_Q(\bar{t})$ can be obtained as $(\bar{t} - t_{q'}) \cdot -\gamma/\alpha + \gamma$. Following the definition of $\tilde{t}$, we have $\tilde{t} + T_Q(\tilde{t}) = \bar{t}$, then

$$\tilde{t} + \frac{\beta}{\alpha - \beta} \cdot (\tilde{t} - t_q) = \bar{t} \tag{6}$$

$$\tilde{t} + \frac{-\gamma}{\alpha + \gamma} \cdot (\tilde{t} - t_{q'}) = \bar{t} \tag{7}$$

Next, since T_w is defined as the average (fixed) time length that each of N bulk ships staying at a berth, the queuing time length for $[t_q, t_{q'}]$ can be obtained as

$$t_{q'} - t_q = T_w \cdot (N - 1) \tag{8}$$

Solving (6)–(8), we then obtain three arrival time values $\tilde{t}$, t_q and $t_{q'}$ as follows:

$$\tilde{t} = \bar{t} - \frac{\beta \cdot \gamma}{\alpha(\beta + \gamma)} \cdot T_w \cdot (N - 1) \tag{9}$$

$$t_q = \bar{t} - \frac{\gamma}{\beta + \gamma} \cdot T_w \cdot (N - 1) \tag{10}$$

$$t_{q'} = \bar{t} + \frac{\beta}{\beta + \gamma} \cdot T_w \cdot (N - 1) \tag{11}$$

Because all bulk ships have the same cost in equilibrium, by substituting $T_Q(\tilde{t}) = \bar{t} - \tilde{t}$ and (9) into (1), or substituting $T_Q(t_q) = 0$ and (10) into (2), or substituting $T_Q(t_{q'}) = 0$ and (11) into (3), the equilibrium total cost per bulk ship during $[t_q, t_{q'}]$ can be obtained as

$$ETC = \frac{\beta \cdot \gamma}{\beta + \gamma} \cdot T_w \cdot (N - 1) + w \cdot (t^* - \bar{t}) \tag{12}$$

$\Delta t_q a t_{q'}$ in Fig. 2 represents the queuing cost, $\alpha \cdot T_Q(t)$, in the nontoll equilibrium. Based on (4) and (5), slopes of $\overline{t_q a}$ and $\overline{a t_{q'}}$, can be easily obtained as $\alpha\beta/(\alpha - \beta)$ and $-\alpha\gamma/(\alpha + \gamma)$, respectively.

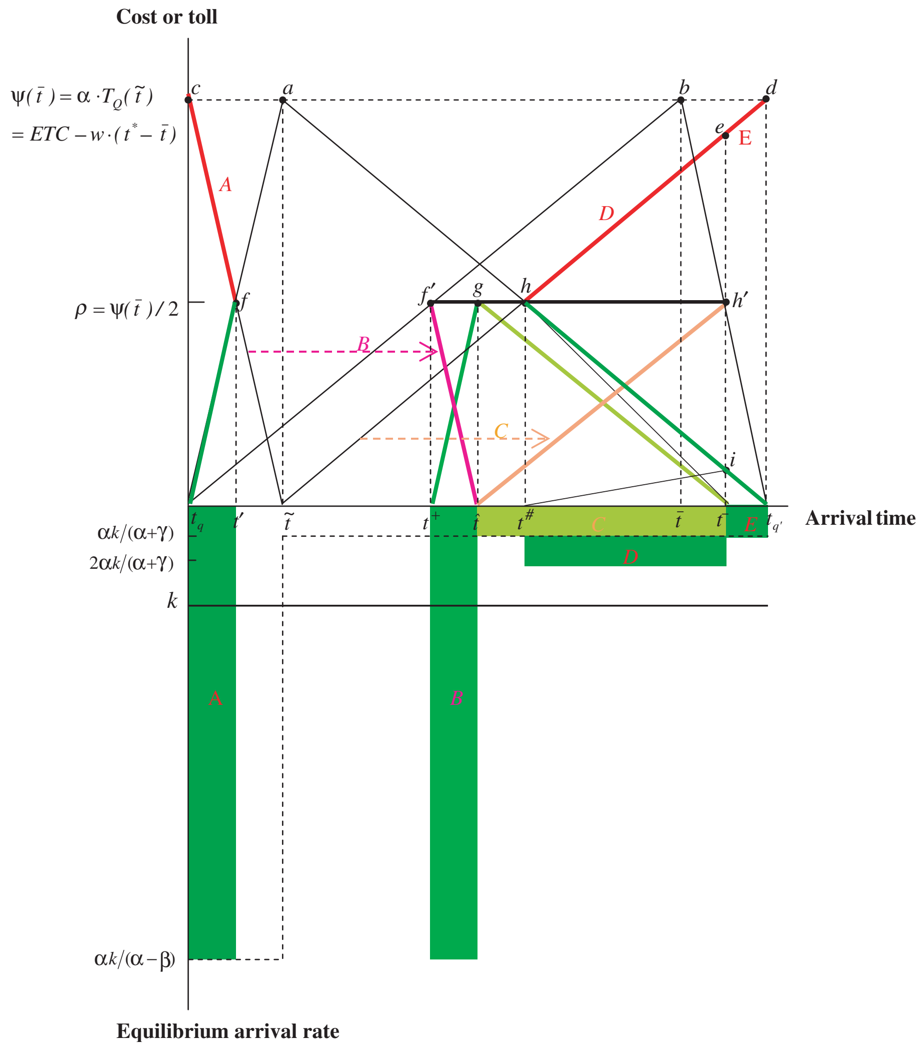

Fig. 2. Equilibrium costs and arrival rate distributions under the optimal single step toll scheme

Next we consider the optimal time-varying toll scheme to bulk ships for a queuing port. The optimal time-varying toll is defined as a series of tolls that will completely eliminate the loss of queuing times without making ship owners worse off than they would be in the nontoll equilibrium. In order to attain such an objective, it is necessary to impose a series of tolls, $\psi(t)$, that results in $T_Q(t)=0$ and $TC(t)=ETC$ for all t in (1), (2) and (3). Then we obtain a series of the optimal time-varying toll as

$$TC(\bar{t}) = \alpha \cdot T_Q(\bar{t}) + w \cdot (t^* - \bar{t}) + \psi(\bar{t}) = ETC$$

$$\psi(\bar{t}) = ETC - w \cdot (t^* - \bar{t}) = \frac{\beta \cdot \gamma}{\beta + \gamma} \cdot T_w \cdot (N-1) \quad (13)$$

$$\begin{aligned} TC(t) &= \alpha \cdot T_Q(t) + w \cdot \left(t^* - \bar{t}\right) + \beta \cdot [t^* - (t + T_Q(t) \\ &\quad + (t^* - \bar{t}))] + \psi(t) \\ &= ETC \end{aligned}$$

$$\begin{aligned} \psi(t) &= ETC - w \cdot \left(t^* - \bar{t}\right) - \beta \cdot (\bar{t} - t) \\ &= \frac{\beta \cdot \gamma}{\beta + \gamma} \cdot T_w \cdot (N-1) - \beta \cdot (\bar{t} - t), \\ &\quad \text{for } t_q \le t < \bar{t} \end{aligned} \quad (14)$$

$$\begin{aligned} TC(t) &= \alpha \cdot T_Q(t) + w \cdot \left(t^* - \bar{t}\right) + \gamma \cdot [(t + T_Q(t) \\ &\quad + (t^* - \bar{t})) - t^*] + \psi(t) \\ &= ETC \end{aligned}$$

$$\begin{aligned} \psi(t) &= ETC - w \cdot \left(t^* - \bar{t}\right) - \gamma \cdot (t - \bar{t}) \\ &= \frac{\beta \cdot \gamma}{\beta + \gamma} \cdot T_w \cdot (N-1) - \gamma \cdot (t - \bar{t}), \\ &\quad \text{for } \bar{t} \le t < t_{q'} \end{aligned} \quad (15)$$

The shape of the optimal time-varying toll scheme, $\psi(t)$, is shown as $\Delta t_q b t_{q'}$ in Fig. 2. Based on (14) and (15), slopes of $\overline{t_q b}$ and $\overline{b t_{q'}}$ are β and $-\gamma$, respectively. The optimal time-varying toll scheme has continuously changeable charges throughout the queuing period $[t_q, t_{q'}]$. The maximum optimal time-varying toll is located at $\bar{t}(=\text{ETA})$ as shown in (13). This is reasonable because ship owners are willing to pay the highest optimal time-varying toll to match the on-time schedule (ETA and ETD) without incurring any early or late costs. Since the two triangles ($\Delta t_q a t_{q'}$ and $\Delta t_q b t_{q'}$) in Fig. 2 have the same area, the total optimal time varying toll completely replace all bulk ships' queuing costs in the nontoll equilibrium.

III. The Optimal Single Step Toll Scheme

The optimal time-varying toll is capable of eliminating queuing time completely, but has practical difficulties because it requires continuously changeable charges. Therefore a step toll scheme, which is first developed by Laih (1994), has been considered as an alternative to reduce queuing time. The step toll scheme, inscribed in the optimal time-varying toll triangle, is designed to make the toll payers no worse off than they would be in the nontoll equilibrium. As shown in Fig. 2, the optimal single step toll, $\rho = \psi(\bar{t})/2$, inscribed within the optimal time-varying toll, $t_q b t_{q'}$, is applied at t^+ and lifted at t^-, and the toll revenue is shaped as $t^+ f' h' t^-$. This is the maximum toll revenue in all single step toll cases because the toll level is just half of the highest optimal time-varying toll. Therefore the optimal single step toll scheme removes the largest proportion of the total queuing time in all single step toll cases and make ship owners no worse off than they would be in the nontoll equilibrium. Under this optimal single step toll scheme, $\hat{t}$ in Fig. 2 is defined as a bulk ship's arrival time at the anchorage which allows the berthing time just the same as ETA after queuing. It is reasonable that $\hat{t}$ under the optimal single step toll scheme is later than $\tilde{t}$ under the nontoll equilibrium.

In Fig. 2, t' is defined as the start time when no bulk ship arrives at the anchorage until t^+ under the optimal single step toll scheme. According to this pricing scheme, the queuing cost to the first bulk ship that will pay the toll, ρ, at t^+ is zero. It is then clear that no bulk ship arrives at the anchorage from t' until t^+, and the length of the time period $[t', t^+)$ is equal to ρ/α. Consequently the last untolled bulk ship before t^+ must arrive at the anchorage ρ/α earlier than the first tolled bulk ship at t^+.

On the other hand, $t^\#$ in Fig. 2 is defined as the start time when the first bulk ships arrive at the anchorage but decide not to notify the port officer for berthing until t^- in order to avoid paying the toll. Also according to this pricing scheme, the queuing cost to the last bulk ship that will pay the toll ρ before t^- is zero. This is impossible unless there are a mass of arrived bulk ships waited willingly at the anchorage from $t^\#$ until t^- to avoid being tolled. These speculators are ready to enter the berth free once the toll is lifted on t^-. Consequently, the length of the time period $[t^\#, t^-]$must be equal to ρ/α.

Note that t_q is now assumed to locate on the origin (i.e. $t_q = 0$) in Fig. 2 for the purpose of simplifying computation to all arrival time values without losing the generality. Detailed computations to the toll level and arrival times values appeared in Fig. 2 are shown as follows:

$$\rho = \frac{1}{2}\psi(\bar{t}) = \frac{1}{2}\frac{\beta \cdot \gamma}{(\beta+\gamma)} \cdot T_w \cdot (N-1),$$

$$t_{q'} = t_q + T_w \cdot (N-1) = T_w \cdot (N-1)$$

$$\tilde{t} = t_q + \overline{t_q \tilde{t}} = 0 + \frac{ETC - w \cdot (t^* - \bar{t})}{\frac{\alpha \cdot \beta}{\alpha - \beta}}$$

$$= \frac{\gamma \cdot (\alpha - \beta)}{\alpha \cdot (\beta + \gamma)} \cdot T_w \cdot (N-1)$$

$$\bar{t} = \tilde{t} + T_Q(\tilde{t}) = \tilde{t} + \frac{ETC - w \cdot (t^* - \bar{t})}{\alpha}$$

$$= \frac{\gamma}{\beta + \gamma} \cdot T_w \cdot (N-1)$$

$$\hat{t} = \bar{t} - T_Q(\hat{t}) = \bar{t} - \frac{\rho}{\alpha} = \frac{\gamma \cdot (2\alpha - \beta)}{2\alpha \cdot (\beta + \gamma)} \cdot T_w \cdot (N-1),$$

$$t^+ = t_q + \overline{t_q t^+} = 0 + \frac{\rho}{\beta} = \frac{\gamma}{2(\beta+\gamma)} \cdot T_w \cdot (N-1)$$

$$t' = t^+ - \frac{\rho}{\alpha} = \frac{\gamma \cdot (\alpha - \beta)}{2\alpha \cdot (\beta + \gamma)} \cdot T_w \cdot (N-1),$$

$$t^- = t_{q'} - \overline{t^- t_{q'}} = t_{q'} - \frac{\rho}{\gamma} = \frac{\beta + 2\gamma}{2(\beta+\gamma)} \cdot T_w \cdot (N-1)$$

$$t^\# = t^- - \overline{t^\# t^-} = t^- - \frac{\rho}{\alpha}$$

$$= \frac{2\alpha\gamma + \alpha\beta - \beta\gamma}{2\alpha(\beta+\gamma)} \cdot T_w \cdot (N-1)$$

IV. Equilibrium Results Under the Optimal Single Step Toll Scheme

This section illustrates some equilibrium results, as shown in columns [1]–[5] of Table 1, for all arrival intervals under the optimal single step toll scheme to bulk ships. Note that a blanket arrival time interval

Table 1. Equilibrium results and arrival time adjustments under the optimal single step toll scheme

[1] Groups (Arrival times at the anchorage)	[2] Types (a)Toll payers or not (b)Departure patterns	[3] EQC: $\alpha \cdot T_Q(t)$ *EOC*: $ETC - EQC - \rho$	[4] EAR : $k + \frac{d(k \cdot T_Q(t))}{t}$	[5] = [4] × [1] Number of arrivals at the anchorage	[6] Arrival time adjustments
A, $t \in [t_q, t']$ (No Toll Period)	(a)Toll free (b)Early departures	$(\overline{t_q f})\frac{\alpha\beta \cdot t}{\alpha - \beta}$	$\frac{\alpha k}{\alpha - \beta}$	$\frac{\gamma k}{2(\beta+\gamma)} \cdot \varepsilon$	$[t_q, t')$ $(\because \overline{cf} = \overline{cf})$
		$(\overline{cf})\frac{-\alpha\beta \cdot t}{\alpha - \beta} + \frac{\beta\gamma}{\beta+\gamma} \cdot \varepsilon$			
$t \in [t', t^+]$ (No Toll Period)	None	0	0	0	None
B, $t \in [t^+, \hat{t}]$ (Tolled Period)	(a).Toll payers (b).Early departures	$(\overline{t^+ g})\frac{\alpha\beta \cdot t}{\alpha - \beta} - \frac{\alpha\beta\gamma}{2(\alpha-\beta)(\beta+\gamma)} \cdot \varepsilon$	$\frac{\alpha k}{\alpha - \beta}$	$\frac{\gamma k}{2(\beta+\gamma)} \cdot \varepsilon$	$[t', \tilde{t}) \rightarrow [t^+, \hat{t})$ $(\because \overline{f\tilde{t}} = \overline{f'\hat{t}})$
		$(\overline{f'\hat{t}})\frac{-\alpha\beta \cdot t}{\alpha - \beta} + \frac{\beta\gamma(2\alpha-\beta)}{2(\alpha-\beta)(\beta+\gamma)} \cdot \varepsilon$			
C, $t \in (\hat{t}, t^-)$ (Tolled Period)	(a)Toll payers (b)Late departures	$(\overline{gt^-})\frac{-\alpha\gamma \cdot t}{\alpha+\gamma} + \frac{\alpha\gamma(\beta+2\gamma)}{2(\alpha+\gamma)(\beta+\gamma)} \cdot \varepsilon$	$\frac{\alpha k}{\alpha+\gamma}$	$\frac{\beta k}{2(\beta+\gamma)} \cdot \varepsilon$	$[\tilde{t}, t^\#) \rightarrow [\hat{t}, t^-)$ $(\because \overline{\tilde{t}h} = \overline{\hat{t}h'})$
		$(\overline{\hat{t}h'})\frac{\alpha\gamma \cdot t}{\alpha+\gamma} - \frac{\gamma^2(2\alpha-\beta)}{2(\alpha+\gamma)(\beta+\gamma)} \cdot \varepsilon$			
D, $t \in (t^\#, t^-)$ (Tolled Period)	(a)Toll free (speculators) (b)Late departures	$(\overline{hi})\frac{-\alpha\gamma \cdot t}{\alpha+\gamma} + \frac{\alpha\gamma}{\alpha+\gamma} \cdot \varepsilon$	$\frac{\alpha k}{\alpha+\gamma}$	$\frac{\beta\gamma \cdot k}{2(\alpha+\gamma)(\beta+\gamma)} \cdot \varepsilon$	$[t^\#, t^-)$ $(\because \overline{he} = \overline{he})$
		$(\overline{he})\frac{\alpha\gamma \cdot t}{\alpha+\gamma} - \frac{\gamma^2(\alpha-\beta)}{\alpha+\gamma} \cdot \varepsilon$			
E, $t \in (t^-, t_{q'})$ (No Toll Period)	(a)Toll free (b)Late departures	$(\overline{it_{q'}})\frac{-\alpha\gamma \cdot t}{\alpha+\gamma} + \frac{\alpha\gamma}{\alpha+\gamma} \cdot \varepsilon$	$\frac{\alpha k}{\alpha+\gamma}$	$\frac{\alpha\beta k}{2(\alpha+\gamma)(\beta+\gamma)} \cdot \varepsilon$	$[t^-, t_{q'})$ $(\because \overline{ed} = \overline{ed})$
		$(\overline{ed})\frac{\alpha\gamma \cdot t}{\alpha+\gamma} - \frac{\gamma^2(\alpha-\beta)}{\alpha+\gamma} \cdot \varepsilon$			

Note: $\varepsilon = T_w \cdot (N-1), k = N/T_w \cdot (N-1)$.

$[t', t^+)$ is existed in Table 1 because no bulk ship arrives at the anchorage during this time period that mentioned before. Bulk ships of groups B, C and D arrive at the anchorage during the tolled period $[t^+, t^-)$. Except for group D, speculators, escaping from being tolled, groups B and C pay the toll to enter the berth. Groups A and E do not need to pay the toll because they arrive at the anchorage during the no toll periods. In addition, only groups A and B will be alongside the berth earlier than ETA because they arrive at the anchorage before $\hat{t}$. Consequently only groups A and B are early departures because they will leave the berth (depart from the queuing port) earlier than ETD. These results are arranged as columns [1]–[2] in Table 1.

Since equilibrium will be achieved as long as all bulk ships have the same total cost throughout the queuing period, there are two kinds of equilibrium conditions for the early and late departures. One is $TC(t) = TC(t_q)$ for groups A and B of the early departure, and the other is $TC(t) = TC(t_{q'})$ for groups C, D and E of the late departure. These equilibrium conditions then can be expressed as follows:

$$\alpha \cdot T_Q(t) + w \cdot T_w + \beta \cdot \left[t^* - \left(t + T_Q(t) + T_w\right)\right] = \beta \cdot \bar{t} + w \cdot (t^* - \bar{t}), \quad t_q \le t \le t' \tag{16}$$

$$\alpha \cdot T_Q(t) + w \cdot T_w + \beta \cdot \left[t^* - \left(t + T_Q(t) + T_w\right)\right] + \rho = \beta \cdot \bar{t} + w \cdot (t^* - \bar{t}), \quad t^+ \le t < \hat{t} \tag{17}$$

$$\alpha \cdot T_Q(t) + w \cdot T_w + \gamma \cdot \left[\left(t + T_Q(t) + T_w\right) - t^*\right] + \rho = \gamma \cdot (t_{q'} - \bar{t}) + w \cdot (t^* - \bar{t}), \quad \hat{t} < t < t^- \tag{18}$$

$$\alpha \cdot T_Q(t) + w \cdot T_w + \gamma \cdot \left[\left(t + T_Q(t) + T_w\right) - t^*\right] = \gamma \cdot (t_{q'} - \bar{t}) + w \cdot (t^* - \bar{t}), \quad t^\# \le t < t^- \text{ or } t^- \le t \le t_{q'} \tag{19}$$

The above equilibrium conditions (16)–(19) are established to groups A, B, C and D (=E), respectively. The values of $TC(t_q)$ and $TC(t_{q'})$ are $\beta \cdot \bar{t} + w \cdot (t^* - \bar{t})$ and $\gamma \cdot (t_{q'} - \bar{t}) + w \cdot (t^* - \bar{t})$, respectively, because $t_q = 0$ and $T_Q(t_q) = T_Q(t_{q'}) = 0$.

Equilibrium queuing costs at the anchorage $(EQC : \alpha \cdot T_Q(t))$ to groups A–E, listed in column [3] of Table 1 are obtained based on (16)–(19). As shown in Fig. 2, the equilibrium queuing costs to groups A–E under the optimal step toll scheme are green lines $\overline{t_qf}$, $\overline{t^+g}$, $\overline{gt^-}$, $\overline{hi}$ and $\overline{it_{q'}}$, respectively. The slope of $\overline{t_qf}$ and $\overline{t^+g}$ for all early departures is $\alpha\beta/(\alpha - \beta)$, which is the same as the slope of the equilibrium queuing cost $(\overline{t_qa})$ to all early departures in the nontoll case. Note that there is no green lines of equilibrium queuing costs through the arrival period $[t', t^+)$ since no bulk ship arrives at the anchorage during this period. In addition, the length of the queue will be reduced to zero at t^+ because of $T_Q(t') = \rho/\alpha = t^+ - t'$. On the other hand, the slope of $\overline{gt^-}$, $\overline{hi}$ and $\overline{it_{q'}}$ for all late departures is $-\alpha\gamma/(\alpha + \gamma)$, which is the same as the slope of the equilibrium queuing cost $(\overline{at_{q'}})$ to all late departures in the nontoll case. Note that group D ships' equilibrium queuing costs incurred before and after t^- in Fig. 2 are $\overline{ht^-}$ and $\overline{t^\# i}$, respectively. The former is the decreasing equilibrium queuing costs, from h to zero, to those speculators who avoid paying the toll during the tolled period $[t^\#, t^-)$. The latter is their increasing equilibrium queuing costs, from zero to i. Consequently, the total equilibrium queuing cost for group D ships is $\overline{hi} (= \overline{ht^-} + \overline{t^\# i})$.

In Fig. 2, the total equilibrium queuing cost in the original nontoll case is $\Delta t_q a t_{q'}$, and the total equilibrium queuing cost under the optimal single step toll scheme is composed of $\Delta t_q f t'$, $\Delta t^+ g t^-$ and $\Delta t^\# h t_{q'}$. Because $\Delta t^+ g t^-$ can be moved to Δfah due to the same area, it is then clear that the effect of the optimal Single step toll on queuing reduction is just half of the total queuing cost in the nontoll equilibrium.

The equilibrium arrival rates (EAR) for groups A–E in column [4] of Table 1 are obtained by using the corresponding equilibrium queuing time $(T_Q(t))$. The lower part of Fig. 2 shows the EAR distributions varying with arrival times for the nontoll, optimal time-varying toll and optimal single step toll cases. These cases are shown as the dotted line, straight line and the shadowed areas, respectively.

In the nontoll case, the equilibrium queuing costs $(\alpha \cdot T_Q(t))$ for all early and late arrivals in Fig. 2 are $\overline{t_qa}$ and $\overline{at_{q'}}$, respectively, and the slopes for the former and latter are $\alpha\beta/(\alpha - \beta)$ and $-\alpha\gamma/(\alpha + \gamma)$, respectively. Accordingly, the marginal arrival rate $(= \mathrm{d}(k \cdot T_Q(t))/\mathrm{d}k)$ for the early and late arrivals are $\beta k/(\alpha - \beta)$ and $-\gamma k/(\alpha + \gamma)$, respectively. Where k is defined as the average number of arrived bulk ships at the anchorage per hour during the queuing period $[t_q, t_{q'}]$. Then $k = N/(T_w \cdot (N - 1))$. The EAR for the early and late arrivals in the nontoll case are therefore equal to $\alpha k/(\alpha - \beta) (= k + (\beta k)/(\alpha - \beta))$ during $[t_q, \tilde{t})$ and $\alpha k/(\alpha + \gamma) (= k - (\gamma k)/(\alpha + \gamma))$ during $[\tilde{t}, t_{q'})$, respectively. (see the dotted line area in Fig. 2). Next, the EAR in the optimal time-varying case is always equal to k because $T_Q(t) = 0$ for all t. (see the straight line area in Fig. 2).

Equilibrium arrival rates in the optimal step toll case is somewhat more complicated than the previous two cases. First, because the equilibrium queuing cost $\overline{t_qf}$ for group A coincides $\overline{t_qa}$ during $[t_q, t')$ in Fig. 2, EAR for group A during this arrival time period is the same as that in the nontoll case, and shown as the shadowed area A. Secondly, since there are no arrivals during $[t', t^+)$, EAR is zero for this time period. Thirdly, because the slopes of equilibrium queuing costs $\overline{t^+g}$ for group B and $\overline{gt^-}$ for group C in Fig. 2 are the same as the slope of $\overline{t_q a}$ and $\overline{at_{q'}}$, respectively, EAR for group B during $[t^+, \hat{t})$ and EAR for group C during $[\hat{t}, t^-)$ are $\alpha k/(\alpha - \beta)$ and $\alpha k/(\alpha + \gamma)$, respectively. The former is shown as the shadowed area B, and the latter is shown as the shadowed area C. Note that there exist group D bulk ships that arrived during the tolled period $[t^\#, t^-)$ but decided to enter the berth free after t^-. It is clear that their arrival time period $[t^\#, t^-)$ overlaps with group C. Because the equilibrium queuing cost $\overline{hi}(= \overline{ht^-} + \overline{t^\# i})$ for group D has the same slope with $\overline{at_{q'}}$, EAR for group D during $[t^\#, t^-)$ is also equal to $\alpha k/(\alpha + \gamma)$. The total EAR for both groups C and D during $[t^\#, t^-)$ in Fig. 2 therefore becomes $2\alpha k/(\alpha + \gamma)$. Finally, since the equilibrium queuing cost $\overline{it_{q'}}$ for group E coincides $\overline{at_{q'}}$ during $[t^-, t_{q'}]$, EAR for group E during this arrival time period is also $\alpha k/(\alpha + \gamma)$.

The numbers of arrived bulk ships listed in column [5] of Table 1 are computed by multiplying the lengths of arrival time intervals and corresponding values of EAR together. Arrival time intervals are shown as column [1], and the lengths for these intervals can be obtained by using the related departure time values shown in the end of Section III. Values of EAR for these intervals have already shown in column [4]. From column [5], it is clear that numbers of the early (groups A and B) and late (groups C, D and E) arrived bulk ships are equal to $[(\gamma k)/(\beta + \gamma)] \cdot T_w \cdot (N - 1)$ and $[(\beta k)/(\beta + \gamma)] \cdot T_w \cdot (N - 1)$, respectively, and both are independent of α.

V. Bulk Ship Owners' Decisions of Arrival Time Adjustments

Other equilibrium costs for a bulk ship staying at the berth ($EOC = ETC - EQC - \rho$) in the optimal single step toll case must be the same as that in the original nontoll case to maintain the equilibrium total cost, $ETC = [(\beta \cdot \gamma)/(\beta + \gamma)] \cdot T_w \cdot (N - 1) + w \cdot (t^* - \tilde{t}) = \psi(\tilde{t})$ because the optimal step toll derived from our model is simply the money cost to the tolled bulk ships that require to save the same amount of queuing costs. For this purpose, bulk ship owners' decisions of arrival time adjustment can be investigated by '*the invariant EOC principle*'.

In Table 1, the contents of the equilibrium total cost to groups A–D(= E) bulk ships under the optimal step toll scheme can be shown as $\alpha \cdot T_Q(t) + w \cdot T_w + \beta \cdot [t^* - (t + T_Q(t) + T_w)]$, $\alpha \cdot T_Q(t) + w \cdot T_w + \beta \cdot [t^* - (t + T_Q(t) + T_w)] + \rho$, $\alpha \cdot T_Q(t) + w \cdot T_w + \gamma \cdot [(t + T_Q(t) + T_w) - t^*)] + \rho$ and $\alpha \cdot T_Q(t) + w \cdot T_w + \gamma \cdot [(t + T_Q(t) + T_w) - t^*)]$, respectively. Since all results of equilibrium queuing costs to bulk ships at the anchorage (EQC) under the optimal single step toll scheme have been shown in column [3] of Table 1, other equilibrium costs to bulk ships staying at the berth (EOC) required to achieve the equilibrium total cost, ETC, can be easily obtained as shown in the same column. EOC to groups A–E bulk ships under the optimal single step toll scheme are drawn as the red lines $\overline{cf}$, $\overline{f\hat{t}}$, $\overline{\hat{t}h'}$, $\overline{he}$ and $\overline{ed}$, respectively in Fig. 2. The slope of $\overline{cf}$ and $\overline{f\hat{t}}$ to all early departures is identical and equal to $-\alpha\beta/(\alpha - \beta)$. This is the same as the slope of $\overline{c\tilde{t}}$ that represents all early departures' EOC in the nontoll case. On the other hand, the slope of $\overline{\hat{t}h'}$, $\overline{he}$ and $\overline{ed}$ to all late departures is identical and equal to $\alpha\gamma/(\alpha + \gamma)$. This is also the same as the slope of $\overline{\tilde{t}d}$ that represents all late departures' EOC in the nontoll case.

Bulk ship owners' decisions of arrival time adjustments under the optimal step toll scheme are shown as column [6] in Table 1. First, group A ships will not adjust their original arrival times in the nontoll case when the port is priced with the optimal single step toll, because $\overline{cf}$ in both the nontoll and optimal single step toll cases coincide during the arrival period $[t_q, t')$. Secondly, because $\overline{f\hat{t}}$ in the optimal single step toll case and $\overline{f\tilde{t}}$ in the nontoll case are two identical and parallel lines, all group B ships that originally arrive during the period $[t', \tilde{t})$ in the nontoll case will adjust their arrivals to the period $[t^+, \hat{t})$ in the optimal single step toll case. Similarly, because $\overline{\hat{t}h'}$ in the optimal single step toll case and $\overline{\tilde{t}h}$ in the nontoll case are two identical and parallel lines, all group C ships that originally arrive during the period $[\tilde{t}, t^\#)$ in the nontoll case will adjust their arrivals to the period $[\hat{t}, t^-)$ in the optimal single step toll case. Thirdly, because $\overline{he}$ in both the nontoll and optimal single step toll cases coincide during the arrival time period $[t^\#, t^-)$, group D ships will not change their original arrival times in the nontoll case when the port is priced with the optimal single step toll. Since $[t^\#, t^-)$ exists within $[\hat{t}, t^-)$, groups C and D ships arrive simultaneously during $[t^\#, t^-)$. Finally, because $\overline{ed}$ in both the nontoll and optimal single step toll cases coincide during the arrival time period $[t^-, t_{q'}]$, group E ships will not change their original arrival times in the nontoll case when the port is priced with the optimal single step toll.

It is clear from the above outcomes that bulk ship owners who choose the same arrival times at the anchorage as they did in the original nontoll equilibrium case are not the toll payers in the tolled case. As shown in Fig. 2, these bulk ships are groups A, D and E. On the other hand, other groups of bulk ships that postpone their original arrival times at the anchorage are the toll payers. As shown in Fig. 2, $[t', \tilde{t}) \rightarrow [t^{+}, \hat{t})$ for group B ships, and $[\tilde{t}, t^{\#}) \rightarrow [\hat{t}, t^{-})$ for group C ships.

VI. Conclusions

This paper considered a pricing model for bulk ships anchoring a queuing port. According to bulk ships' departure patterns at a queuing port, we derived the non-toll equilibrium total cost (*ETC*) per bulk ship during the queuing period. We also developed a series of optimal time-varying tolls that eliminate the loss of queuing times without making ship owners worse off than they would be in the non-toll equilibrium. Because the optimal time-varying toll scheme has practical difficulties, the optimal single step toll inscribed in the optimal time-varying toll is established in this paper as an alternative pricing scheme to bulk ships at a queuing port.

This paper has shown all arrival time values, equilibrium conditions, equilibrium queuing costs at the anchorage (*EQC*), equilibrium arrival rates (EAR) and other equilibrium costs for a bulk ship staying at a berth (*EOC*) under the optimal single step toll scheme. By following the invariant *EOC* principle to achieve the equilibrium purpose, this paper provided a framework to predict bulk ship owners' decisions of arrival time adjustment under the optimal single step toll scheme. We have found that bulk ship owners who choose the same arrival times at the anchorage as they did in the original non-toll equilibrium case are not the toll payers under the optimal single step toll scheme. The other part of bulk ships that postpone their original arrival times at the anchorage are the toll payers under the optimal single step toll scheme.

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